# UNIVERSITY OF NAIROBI

## Departments of Building and Land Economics

# Elements of Theory of Structures

# (BCM/BQS/BLE 110)

Teaching notes

By Sixtus Kinyua Mwea

October 10, 2012

Syllabus

1 Consideration of various aspects of forces;

1.1 Equations of equilibrium

1.2 Free body diagrams

2 Simple stress systems

2.1 Direct compression

2.2 Tension

2.3 Shear

3 Pin jointed structures

3.1 Trusses

3.2 Arches

4 Analysis of beams

4.1 Simple bending moment and shear force diagrams

## Table of contents

**Chapter One**

# Introduction

## 1.1 Need for Engineering Courses to Building and Land Economist students

Theory of structures courses covered in the syllabus is an introduction to structural engineering. The professionals in quantity surveying, construction management and land economics require appreciating basic structural engineering because their work involves measurement and valuation of structures and in particular buildings. It is important at the outset to say that basic consultation between the structural engineer and these professionals is necessary and the introduction courses which you get introduced to can not replace the place of the structural engineer in the professional team.

The introductory course will however enable the land and building economist to read engineering drawings and engineering reports so that they can draw meaningful conclusions. In addition they will be able to participate in the team of professionals in the building team in the case of the building economists. In some occasions when the nature of a project is not complex the client may seek the services of the land or building economist without the need to hire out a structural engineer. In this case the hired consultant should appreciate the structural status of the project.

It is worth noting that the knowledge of quantity surveying and costing is sometimes limited to the civil and structural engineers. This weak link between the professionals has sometimes led to conflict in project management. It is not uncommon for the value and cost works which are not certified by the structural engineer

## 1.2 Structural Engineering

A structure is created by the designers to serve a definite purpose. The structures we are thinking about are many and varied. They include buildings, bridges, water storage facilities etc. The structures should be built in materials which are carefully designed and selected so that they can resist the loads that act on them. Determination of the loads is complicated by variation of architectural design, variety of materials and location of the structure. In addition the loads on structures change from time to time and in some instances the change is rapid with time.

For architectural structures the principal and most important loads are self and imposed. The self loads do not change with time. These loads are also called dead loads. The imposed loads on the other had result from the usage of the structure. These loads resulting are difficult to determine accurately. They change with time in both magnitude and location on the structure. These loads are also called imposed loads and are obtained from codes of practice (BS 6399). They are determined from experience and statistical analysis. When used for the intended purpose of the structure, the applied imposed loads do not exceed the code prescriptions during the life time of the structure.

Besides the above loads the structures are subject to environmental loads. The principal environmental loads include earthquake and wind loads. Other secondary environmental loads include temperature variation induced loads. Coastal and marine structures will in additional be subjected to wave action.

Structural analysis and design should lead to construction of safe structures. These structures should be safe from collapse, excessive settlement distortion or excessive vibration. Accurate determination of these loads enables the computation of forces in the structural analysis and design. The loads outlined above are described further to enable their determination.

### 1.2.1 Definition of force

Force is described as a measurable and determinable influence inclining a body to motion. In order for a body to move or to cause a change in the movement of a body, a force must act upon the body. Newton defined force as the product of mass and the rate of change of velocity.

$$p = ma$$
$$a = (v_1 - v_2)$$

Where $p$ = applied force

$m$= mass of the body

$v_1$ = initial velocity

$v_2$ = final velocity

$t$ = time between the change of velocity from $v_1$ to $v_2$

The loads that generate forces for the design of structures are shown on Table 1.2. They are accelerated to the centre of the earth by gravity. The masses are restrained from accelerating to the centre of the earth by the structural elements. Hence a force is generated. The acceleration to the centre of earth due to gravity at the sea level is 9.81m/sec$^2$. This is rounded up to 10m/sec$^2$

In SI units 1N = 1kg mass * 1m/sec$^2$

For plain concrete the density is 2200kg/m$^3$ = 2200*10 N/m$^3$ =22,000 N/m$^3$ = 22kN/m$^3$

Forces are vector quantities defined in magnitude and direction. For a force acting at a particular point on surface of a structure it is necessary for the magnitude and direction of the force to be known and to be determined. Forces in the same plane are known as coplanar and defined by two items of data namely magnitude and direction only. Non coplanar forces are related to three dimensions. In a majority of structures it is conventional to simplify the loading into a two dimensional problems and have a coplanar force analysis system. This course is limited to coplanar analysis

### 1.2.2 Dead loads

Dead loads are those loads which are related to the mass of the structure. These loads remain static during the usage of the structure. In structures made of heavy materials like concrete and masonry the dead loads form the main structural loads. In additional to the weight of the structures the designer needs to take into account of permanent partitions, finishes and services. Dead loads are computed from known densities of the construction materials and those materials that impose dead load to the structure. The densities are turned into force units by multiplying with gravity acceleration. Table 1.1 shows the densities and unit weights of the commonly used building materials in this country.

In additional to the above loads where the partitions are not shown on an open floor plan an allowance is made equal to 1/3 of the weight of the partitions per metre run. In the case of demountable partitions the equivalent dead weight allowed for in design is 1 $kN/m^2$

**Table 1.1** Dead loads of commonly used materials in structures

| Material | Density ($kg/m^3$) | Material | Density ($kg/m^3$) |
|---|---|---|---|
| Reinforced concrete | 2400 | Compacted soil | 1760 |
| Plain concrete | 2200 | | |
| Structural steel | 7800 | | |
| Wood | 500 | | |
| Nairobi building stone | 2600 | | |
| Thika Machine Dressed stones | 2200 | | |
| **Concrete blocks allowing 25mm plaster** | | | |
| **Material** | **Weight ($kg/m^2$)** | **Material** | **Weight ($kg/m^2$)** |
| 250mm solid | 480 | 150 mm solid | 316 |
| 200mm solid | 400 | 100 mm solid | 220 |
| **Hollow Concrete blocks allowing 25mm plaster** | | | |
| 200mm | 300 | 100mm | 160 |
| 150mm | 270 | | |
| **Hollow Clay blocks allowing 25mm plaster** | | | |
| 150mm | 165 | 100mm | 122 |
| **Screeds** | | | |
| 25mm | 60 | 50 | 120 |
| **Floors** | | | |
| 25mm wood block | 60 | 3mm plastic tiles | 5 |
| 12mm quarry tiles | 30 | 25 mm plaster | 50 |
| **Roofs** | | | |
| Clay roofing tiles | 60 | Asbestos sheets | 17 |
| Galvanized sheets | 12 | Aluminum sheets | 2 |
| Bitumen felt | 15 | | |

### 1.2.3 Imposed loads

These are loads appropriate to the different building or structure usages. They include the movable weights to be carried across the structure including furniture and any equipment to be used in the lifetime of the structure. In addition to their mass weights the loads take into account the dynamic effects of imposed loads. The loads given in the codes include distributed loads which are to be used in all cases where the floor is capable of distributing the load laterally. This is the case of the commonly used concrete floors. As an alternative concentrated loads may be used where the floor is incapable of distributing the loads laterally. These include floors made up of closely spaced beams.

Imposed loads are presented in building codes of practice and vary with occupancy class index which differentiate the usages. The occupancy class indexes include residential, institutional, educational, public assembly, offices, retail storage, industrial storage and vehicular occupancies. For each occupancy, the loads vary with floor usage as can be seen from an extract of commonly used loads (Table 1.2). in accordance with BS 6399.

**Table 1.2 Imposed loads**

Residential occupancy

| **Location** | **Uniformly distributed load (kN/m2)** | **Concentrated loads (kN)** |
|---|---|---|
| **Residential occupancy** | | |
| Self contained units | 1.5 | 1.4 |
| Apartments, guest houses and lodges | | |
| Communal kitchens | 3 | 4.5 |
| Dining rooms, lounges | 2.0 | 2.7 |
| Toilet rooms | 2.0 | |
| Bedrooms | 1.5 | 4.5 |
| Corridors, stairs and hall ways | 3 | 4.5 |
| **Institutional occupancy** | | |
| Dense mobile stacks | 4.8 per metre of stack | 7 |
| Large corridors for wheeled vehicles | 5 | 4.5 |
| Bars | 5 | |
| Corridors, stairs and hall ways | 4 | 4.5 |
| Library and reading rooms | 4 | 4.5 |
| Class rooms and chapels | 3 | 4.5 |
| Bedrooms | 1.5 | 1.8 |
| Stages | 7.5 | 4.5 |
| **Offices occupancy** | | |
| Banking halls | 3 | |
| Offices for general use | 2.5 | 2.7 |

### 1.2.4 Earthquake loads

***The Ritcher and Mercalli scales***

The strength of an earthquake is usually measured on one of two scales, the Modified Mercalli Scale and the Richter Scale. The Mercalli Scale is a rather arbitrary set of definitions based upon what people in the area feel, and their observations of damage to buildings around them. The scale goes from 1 to 12, ( I to XII) or using the descriptive titles of the levels, from Instrumental to Catastrophic Table 1. 3

**Table 1. 3 Modified Mercalli eathrquake scles**

| **Modified Mercalli Scale** | | | |
|---|---|---|---|
| **Intensity** | **Verbal Description** | **Magnitude** | **Witness Observations** |
| I | Instrumental | 1 to 2 | Detected only by seismographs |
| II | Feeble | 2 to 3 | Noticed only by sensitive people |
| III | Slight | 3 to 4 | Resembling vibrations caused by heavy traffic |
| IV | Moderate | 4 | Felt by people walking; rocking of free standing objects |
| V | Rather Strong | 4 to 5 | Sleepers awakened and bells ring |
| VI | Strong | 5 to 6 | Trees sway, some damage from overturning and falling object |
| VII | Very Strong | 6 | General alarm, cracking of walls |
| VIII | Destructive | 6 to 7 | Chimneys fall and there is some damage to buildings |
| IX | Ruinous | 7 | Ground begins to crack, houses begin to collapse and pipes break |
| X | Disasterous | 7 to 8 | Ground badly cracked and many buildings are destroyed. There are some landslides |
| XI | Very Disasterous | 8 | Few buildings remain standing; bridges and railways destroyed; water, gas, electricity and telephones out of action. |
| XII | Catastrophic | 8 or greater | Total destruction; objects are thrown into the air, much heaving, shaking and distortion of the ground |

Whilst this scale is fine if you happen to experience an earthquake in an inhabited area of a developed country, it is of no use whatsoever in the middle of a desert or in any other place without trees, houses and railways! Descriptions such as "Resembling vibrations caused by heavy traffic." depend very much upon the observer having felt heavy traffic in the past. Even then, what one person in a small town considers to be 'heavy' will most certainly differ from what a person living adjacent to a major urban road system would describe as 'heavy'.

Clearly this scale has advantages, but something else is required if we are to be able to compare the magnitude of earthquakes wherever they occur. The Intensity Scale differs from the **Richter Magnitude Scale** in that the effects of any one earthquake vary greatly from place to place, so there may be many **Intensity** values (e.g.: **IV**, **VII**) measured for the same earthquake. Each earthquake, on the other hand, should have only one **Magnitude**, although the various methods of calculating it may give slightly different values (e.g.: **4.5**, **4.6**).

The Richter Scale is designed to allow easier comparison of earthquake magnitudes, regardless of the location.

C.F.Richter was a geologist living and working in California, U.S.A, an area subjected to hundreds of 'quakes every year. He took the existing Mercalli scale and tried to add a 'scientific' scale based on accurate measurements that could be recorded by seismographs ( instruments used to measure vibration) regardless of their global location.

By measuring the speed, or acceleration, of the ground when it suddenly moves, he devised a scale that reflects the 'magnitude' of the shock. The Richter scale for earthquake measurements is logarithmic. This means that each whole number step represents a ten-fold increase in measured amplitude. Thus, a magnitude 7 earthquake is 10 times larger than a 6, 100 times larger than a magnitude 5 and 1000 times as large as a 4 magnitude.

This is an open ended scale since it is based on measurements not descriptions.

An earthquake detected only by very sensitive people registers as 3.5 on his scale, whilst the worst earthquake ever recorded reached 8.9 on the 'Richter Scale'.

When trying to understand the forces of an earthquake it can help to concentrate just upon the up and down movements. Gravity is a force pulling things down towards the earth. This accelerates objects at 9.8 m/s$^2$. To make something, such as a tin can, jump up into the air requires a shock wave to hit it from underneath traveling *faster* than 9.8m/s$^2$. This roughly corresponds to 11 (Very disastrous) on the Mercalli Scale, and 8.1 or above on the Richter Scale. In everyday terms, the tin can must be hit by a force that is greater than that which you would experience if you drove your car into a solid wall at 35 khp (22 mph).

**Table 1. 4** Richter Scale

| **Richter Scale** | **Approximate Acceleration** | **Approximate Mercalli equivalent** |
|---|---|---|
| <3.5 | < 1 cm/s$^2$ | I |
| 3.5 | 2.5 cm/s$^2$ | II |
| 4.2 | | III |
| 4.5 | 10 cm/s$^2$ | IV |
| 4.8 | 25 cm/s$^2$ | V |
| 5.4 | 50 cm/s$^2$ | VI |
| 6.1 | 100 cm/s$^2$ | VII |
| 6.5 | 250 cm/s$^2$ | VIII |
| 6.9 | | IX |
| 7.3 | 500 cm/s$^2$ | X |
| 8.1 | 750 cm/s$^2$ | XI |
| > 8.1 | 980 cm/s$^2$ | XII |

***Earthquake loads for structural analysis***

The effects of earthquake are assed as horizontal loads. They are usually applied at each of the floors level or at the junctions of columns and beams. The values of these loads are taken as a fraction of the building weight W. The building depends on the type of building as follows

***a) Normal type of building***

The dead weight of the building including all walls and partitions

***b) Ware house and storage structures***

The dead weight of the building including all walls and partitions plus 25% of the live loads

***c) Liquid and bulk storage***

The dead weight of the structure and its contents

The value of the horizontal force is determined by multiplying the building weight W by coefficient C which depends on the earthquake zone as shown on Table 1.5

**Table 1.5 Earthquake coefficient C**

| Building type | Storeys | Zone VI | | Zone VII | | Zone VIII-IX | |
|---|---|---|---|---|---|---|---|
| | | Soft ground | Hard ground | Soft ground | Hard ground | Soft ground | Hard ground |
| Frames without shear walls-RC or steel | 4 | NR | NR | NR | NR | 0.09 | 0.07 |
| | 10 | 0.02 | 0.017 | 0.034 | 0.028 | 0.068 | 0.055 |
| | 20+ | 0.019 | 0.015 | 0.030 | 0.025 | 0.060 | 0.050 |
| Frames with shear walls-RC or steel | 4 | NR | NR | NR | NR | 0.067 | 0.056 |
| | 10 | 0.016 | 0.013 | 0.026 | 0.022 | 0.053 | 0.044 |
| | 20+ | 0.015 | 0.012 | 0.024 | 0.020 | 0.048 | 0.040 |
| Reinforced load bearing walls | 4 | 0.022 | 0.018 | 0.043 | 0.036 | NS | NS |
| | 10 | 0.017 | 0.014 | 0.032 | 0.027 | NS | NS |
| | 20+ | NS | NS | NS | NS | NS | NS |

Legend NR – No earthquake design required, NS – the building type not suitable

Figure 1.1 shows the map Kenya indicating the seismic zoning of the country. As can be seen the likelihood of the strongest earthquakes is found in the Rift Valley while North Eastern Kenya and a small portion of Nyanza and Western Provinces.

The design against earthquake depends on the type of the building usage and value.  In addition the type of structure dictates the need for earthquake design. The two considerations are presented below.

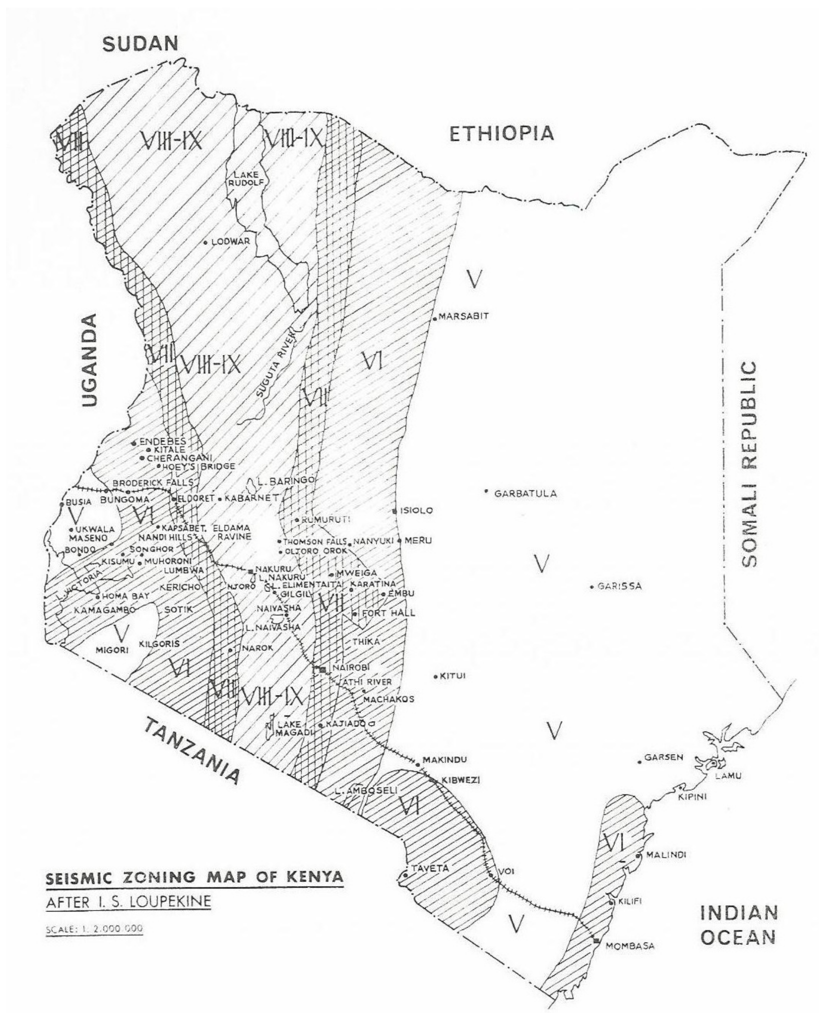



**Figure 1.1  Seismic zoning map of Kenya**

### *Classification by Building usage and value*

- *Class A* – Buildings for Public Assembly and Use

  Government buildings, schools, offices over 4 storeys, flat over 4 storeys etc
- *Class B* – Large buildings for multiple occupation

  Excluding Public assembly and/or vital importance. Includes Hotels and office blocks, restaurants up to 3 storeys and Flats and domestic not exceeding 4 storeys
- *Class C* – Buildings and structures for services and industries

  These includes water supply and treatment plants, power stations, water towers, airport control buildings, dams, water pipelines chimneys, bridges etc
- *Class D* - Domestic buildings within municipalities and townships. Includes low cost houses
- *Class E* - Domestic buildings in rural areas

### *Classification by type of building structure*

- *Framed buildings* - These may be reinforced concrete, structural steel or timber. These may be flexible or non-braced. In this case horizontal forces induced by earthquake or wind are transmitted by the slabs, beams and columns. On the other hand rigid frames are those which take the vertical forces by slabs, beams and columns action. The lateral forces are taken by designed bracings and or shear walls. These may be located at the ends of buildings or in the staircase and lift areas where they are needed for the building usage

- *Load bearing walls* – these are buildings where both vertical and horizontal forces and transmitted by transverse and longitudinal walls built of courses of masonry, brick or concrete blocks.

*General considerations*

- ***Foundations –*** Within a given site foundations should be built on the hardest material available. Foundations should generally be on the same level and tied together. Where there is drastic change in level the foundations the foundations should generally be designed to give uniform pressure. Soft ground needs further investigation to avoid large settlements and consequential cracking of the structure

- ***Shape factor and joints –*** Buildings should as far as possible have simple, uniform and compact configurations. In general the plan shape should be square or rectangular of length to width not greater than 3 to 1. For tall buildings it is advisable to avoid complicated shapes like U E etc. These shapes of buildings vibrate in a complicated

- ***Structural details –*** For load bearing walls sufficient walls in each of the two principal directions should be provided. At each level the floors and roof should be firmly connected with ring beams. Large openings should be restricted unless the area of opening is adequately framed. In case of framed structures some overstress is usually allowed in the design codes. In all cases all infill walls and parapets etc should be adequately tied to the building frame to avoid fall out

Table  shows the seismic design guide in Kenya

| Type of structure | | Zone V | | Zone VI | | Zone VII | | Zone VIII-IX | |
|---|---|---|---|---|---|---|---|---|---|
| | | Seismic design Required | Limit storeys or height | Seismic design Required | Limit storeys or height | Seismic design Required | Limit storeys or height | Seismic design Required | Limit storeys or height |
| Framed structures | Class A | No | No limit | No unless 12 storeys and over | No limit | No unless 6 storeys and over | No limit | Yes | No limit, but design required |
| | Class B | No | 3 storeys for offices – 4 storeys for flats | No | 3 storeys for offices – 4 storeys for flats | No | 3 storeys for offices – 4 storeys for flats | Yes – if 3-4 storeys | 3 storeys for offices – 4 storeys for flats |
| | Class C | No | No limit | No | No limit | Depends on the use an importance and level of damage acceptance. At the discretion of the engineer | | | |
| | Class D | No | 2 storeys | No | 2 storeys | No | 2 storeys | No | 2 storeys |
| Load bearing walls | Class A | No | No limit | Yes | Not over 4 storeys | Yes | Not over 3 storeys | Yes | Not over 2 storeys |
| | Class B | No | 3 storeys for offices – 4 storeys for flats | Yes | 3 storeys for offices – 4 storeys for flats | Yes | 3 storeys for offices – 4 storeys for flats | Yes | Not over 3 storeys |
| | Class C | No | Not over 3 storeys | No | Not over 3 storeys | Load bearing walls are not recommended over 2 storeys | | | |
| | Class D | No | 3 storeys | No | 3 storeys | Yes | 3 storeys | Yes | 2 storeys |
| | Class E | Usually there is no control of domestic buildings in rural areas.   They should however not exceed 3 storeys | | | | | | | |

### 1.2.5 Wind Loads

***Introduction***

Wind loading on buildings depends on the wind velocity, shape and location of the building. The buildings codes prescribe pressures and or suctions for idealized building shapes and locations resulting from wind loads.

***Wind design velocity***

The design velocity is obtained from Equation 1

$V_s = V * S_1 * S_2 * S_3$

Where $V_s$ is the design wind speed

$V$ is the basic wind speed in m/s

$S_1$ is the topographic factor

$S_2$ is the ground roughness factor

$S_3$ is a statistical factor

The basic wind design speed for the design of buildings in Kenya is defined as the speed of a three second gust to be exceeded on the average only once in 50 years in an open country with no obstructions. Typical three gust wind gust velocities for various towns in Kenya for 25 year, 50 year and 100 year return periods is shown on Table

**Table 1. 6 Wind speeds for some towns in Kenya**

| **Town/Return period in years** | **Wind speed (m/s)** | | | **Town/Return period in years** | **Wind speed (m/s)** | | |
|---|---|---|---|---|---|---|---|
| | **25** | **50** | **100** | | **25** | **50** | **100** |
| Mombasa | 25.9 | 28.1 | 30.3 | Nanyuki | 33.1 | 35.9 | 38.7 |
| Kisumu | 41.2 | 44.6 | 48.1 | Nairobi | 24.8 | 26.9 | 29.0 |
| Nakuru | 29.0 | 31.3 | 33.9 | Kericho | 28.6 | 30.9 | 33.4 |

*Source – Metrological Department*

***Topographic factor - $S_1$***

The basic wind speed V takes into account the general level of the site above the sea level. However it does not take into account the local topographical features. Level terrains will attract an $S_1$ factor of 1 while very exposed summits will attract an $S_1$ factor of 1.36.

***Ground roughness factor- $S_2$***

In conditions of strong wind the wind speed usually increases with the height of the building. The rate of increase depends on the ground roughness (Table 1. 7). Additionally the design speed will depend on whether short gusts are taken into account or wind mean speeds are taken into account. For small buildings and elements within the buildings the short gusts are more appropriate while for the larger buildings the mean speeds are more appropriate Table 1. 8 shows the classes in consideration of the building size and cladding elements. The ground roughness factor varies from as low as 0.47 for low surfaces with large obstructions to as high as 1.24 for very high structures in open country with no wind brakes. Table shows the typical values of $S_2$

**Table 1. 7 Ground roughness details**

| Roughness | Details |
|---|---|
| 1 | Open country with no obstructions |
| 2 | Open country with scattered wind breaks such as hedges and walls around fields |
| 3 | Surface covered with numerous obstructions such as forests, towns and outskirts of large cities |
| 4 | Surface covered with numerous obstructions which are over 25 metres high above the ground level as is common at the central business districts of large cities with closely spaced buildings |

**Table 1. 8 Cladding and building size considerations**

| Class | Details |
|---|---|
| A | In consideration of all units of the building – glazing, roofing, and individual members of the unclad structure |
| B | All building and structures where the greatest dimension is less than 50 metres |
| C | All building and structures where the greatest dimension is greater than 50 metres |

**Table 1. 9 Ground roughness, cladding, building size and height above the ground**

| Height (m) | Ground Roughness 1 | | | Ground Roughness 2 | | | Ground Roughness 3 | | | Ground Roughness 4 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Open Country | | | Open country with scattered breaks | | | Country with many wind breaks | | | Surface with large obstructions | | |
| | A | B | C | A | B | C | A | B | C | A | B | C |
| 3 | 0.83 | 0.78 | 0.73 | 0.72 | 0.67 | 0.63 | 0.64 | 0.60 | 0.55 | 0.56 | 0.52 | 0.47 |
| 10 | 1.01 | 0.95 | 0.89 | 0.95 | 0.88 | 0.83 | 0.79 | 0.74 | 0.68 | 0.67 | 0.62 | 0.56 |
| 30 | 1.12 | 1.05 | 0.98 | 1.11 | 1.03 | 0.97 | 1.03 | 0.97 | 0.89 | 0.92 | 0.85 | 0.77 |
| 60 | 1.19 | 1.12 | 1.05 | 1.18 | 1.10 | 1.03 | 1.13 | 1.06 | 0.97 | 1.10 | 1.02 | 0.92 |
| 100 | 1.22 | 1.17 | 1.10 | 1.21 | 1.16 | 1.09 | 1.19 | 1.12 | 1.03 | 1.22 | 1.13 | 1.02 |
| 200 | 1.24 | 1.24 | 1.16 | 1.24 | 1.24 | 1.17 | 1.24 | 1.21 | 1.11 | 1.24 | 1.21 | 1.09 |

***Wind force***

The wind force is subsequently calculated by first estimating the dynamic pressures acting on the building elements. This obtained by multiplying the square of the design velocity by a factor (Equation 1.1). The pressure is then turned into force by application of force coefficients (Equation 1.2)

$$q = 0.613 * V_s^2 \qquad \textbf{1. 1}$$

$$f = cf * q$$

$$F = cf * q * A_e \qquad \textbf{1. 2}$$

Where

$q$ = dynamic pressure in N/m$^2$

$cf$ = force coefficient to be obtained from codes (generally taken as 1.05)

$f$ = pressure in N/m$^2$ to be used in design

$F$ = Force in N to be used in design

$A_e$ = Area of the exposed surface

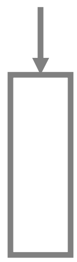
### 1.2.6 Earthquake and wind design in Kenya

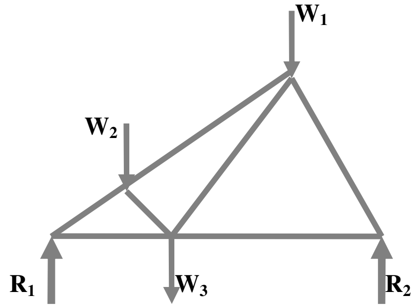
It is unlikely the two effects causing the lateral force effect can occur at the same time. The normal practice is to calculate the lateral force based on the wind and earthquake loads. The design is subsequently based on the more severe effect. The lateral loads for the structures are based on the type of the structural system. However it is more efficient to design structural elements which are efficient in carrying the lateral loads. The efficient elements include specifically designed bracing framework, or shear walls which can be included in the architectural design to give the structure the much needed lateral stability.

### 1.2.7 Loading systems

The basic function of a structural member is to carry the applied loads an transmit them to the other parts of the structure safely. Before any of the various load effects can be considered, the applied loads must be rationalized into a number of orderly systems. The systems must be capable of being defined in mathematical terms without departing from physical reality. For example forces may be applied at a point with no area to enable analyses. Such rationalization when done properly has been known to have acceptable results which are used in structural analyses and design. Figure 1. 2 shows the systems that are encountered in practice

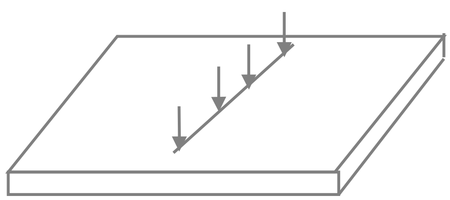
### A) Concentrated loads






***Examples of Applications***
Column loads
Reactions from end of beams
Loads at nodes of frameworks

### B) Knife edge loads

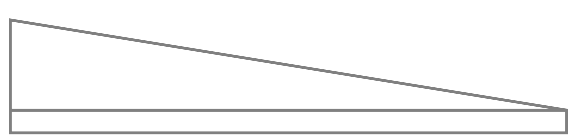


***Example***
Partition walling on a slab

### C) Uniformly distributed loads

***Example***
Load due to self weight of beam

### D) Distributed with linear variation



***Example***
Loads against retaining walls due to retained earth or liquid

**Figure 1. 2 Loading systems**

### 1.2.7 Effects of loading

Having transformed the sometimes complex and irregular applied forces and load into definable load system the designer has to consider how these loads are transmitted by the structure safely. For example in the case of a stretched wire the load is tension which must be transmitted through the length of the wire. The column load is transmitted through the column through compression to lower columns or through to the foundations four load effects can be identified as shown on Table 1.10.

**Table 1.10 Effects of loading**

| Load effect | Symbol | Definition |
|---|---|---|
| Direct effect (Tension or compression) | P or N or W | Acts parallel to the longitudinal axis of the member |
| Shearing force | S or Q | Acts normal to the longitudinal axis |
| Bending moment | M | Turning moment acting in the plane containing the longitudinal axis |
| Torque | T | Turning moment acting normal to the longitudinal axis. Twisting moment. |

Any single or combination of several loads can give rise to one or more of these primary load effects. The members must be designed for these load effects not to distort or fail the members under design. The names of structural member generally reflect the major load effect functions as indicated in Table 1. 11

**Table 1. 11 Members and load effects**

| Type of member | Load effect | Deformation mode | Common orientation |
|---|---|---|---|
| Beams | Bending and shear | Flexural | Horizontal |
| Strut | Direct – compression | Shortening or buckling | Not specific |
| Column | Direct – compression | Shortening or buckling | Vertical |
| Tie | Direct – tension | Lengthening | Not specific |

### 1.2.8 Simple stress systems

***Stress***

When a force is transmitted through a solid body, the body tends to undergo a change in shape. This tendency to deformation is resisted by the internal cohesion of the body. The body is said to be in a state of stress. In other word stress is mobilized force which resists any tendency to deformation. The definition of stress is force transmitted per unit area as shown on Equation 1.1 Load stress relationships are shown on Table

$$Stress = \frac{Force}{Area}(kN/m^2)\ or\ (N/mm^2)$$

$$\sigma = \frac{F}{A}(kN/m^2)\ or\ (N/mm^2) \qquad \textbf{1. 3}$$

Where $\sigma$ = stress (N/mm$^2$ or kN/m$^2$)

$F$= Applied force (N or kN)

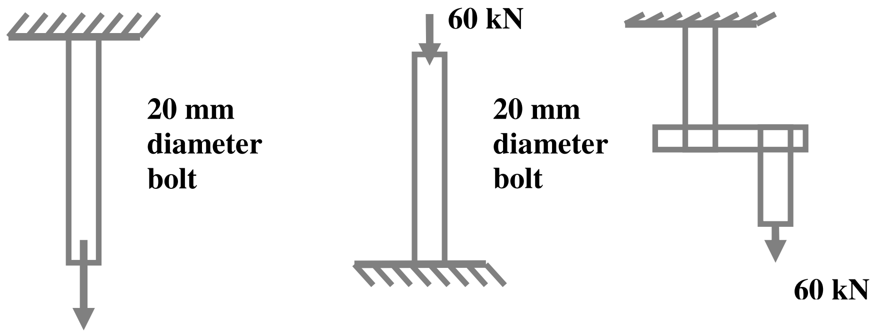
$A$ = Area resisting the Force in mm$^2$ or m$^2$

**Table 1. 12 Load stress relationships**

| Load Effect | Stress induced | |
|---|---|---|
| | Normal | Tangential |
| Direct tension | Tensile | |
| Direct compression | Compressive | |
| Shearing force | | Shearing |
| Bending moment | Tensile and compressive | |
| Torsion | | Shearing |

***Example 1.1***

Calculate the direct axial forces for the bolts in tension compression and shear as shown in Figure 1. **3**a through to c. Additionally calculate the stresses in the bolts




*a) Tension* *b) Compression* *c) Shear*

**Figure 1. 3 Example 1.1**

Solution –

a. Tension force = 60kN

Tension stress

$$\sigma_t = \frac{60}{\pi * 10^2} * 1000$$
$$= 190.7 \quad N/mm^2$$

b. Compression force = 60kN

Compression stress $\sigma_t$

$$\sigma_c = \frac{60}{\pi * 10^2} * 1000$$
$$= 190.7 \quad N/mm^2$$

c. Shearing force = 60kN

Shear stress

$$\sigma_s = \frac{60}{\pi * 10^2} * 1000$$
$$= 190.7 \quad N/mm^2$$

***Strain***

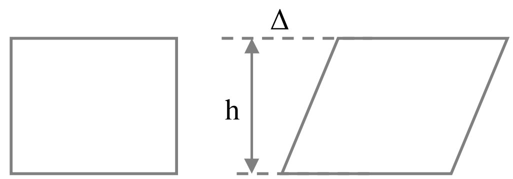
When a force is transmitted through a body the body tends to be deformed. The measure of deformation is strain. Strain is expressed in a way that relates the deformation to the original dimensions of the body as follows.

$$Tensile\ strain = \varepsilon_t = \frac{increase\ in\ lengh}{original\ length} = \frac{\Delta l}{l}$$

$$Compressive\ strain = \varepsilon_t = \frac{decrease\ in\ lengh}{original\ length} = \frac{\Delta l}{l}$$

$$Area\ strain = \varepsilon_A = \frac{Change\ in\ area}{original\ area} = \frac{\Delta A}{A}$$

$$Shearing\ strain = angular\ displacement = \frac{\Delta}{h}$$




***Stress and strain***

Within the range of elasticity which is the working range of structural materials stress is proportional to strain and is related to the Young's modulus as shown in Equation 1.2. Figure 1. 4 shows the relationship of stress and strain of a typical structural material.

***E = σ/ε*** ***1. 4***

Where $E$ = the materials Young's Modulus

$\sigma$ = the stress in the loaded member

$\varepsilon$= the strain in the material

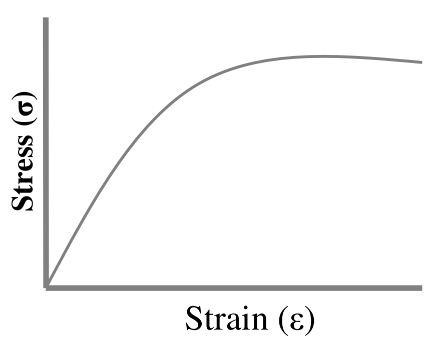



**Figure 1. 4 Stress strain relationship**

***Compound bars***

When a structural member is constructed in such a way that two or more materials are combined in order to take the load simultaneously, and deform equally under axial load the members is referred to as compound bars. Analysis of these bars requires the stresses in the materials are different. The strains can be equated in a compatibility equation. Figure 1. 5 shows a compound bar with two members MA and MB.

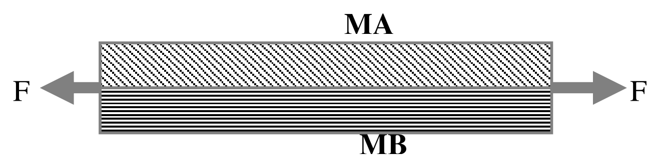



**Figure 1. 5 Compound bar**

Analysis of the forces requires compatibility of strain in materials A and B. Equation 1.3 and 1.4 are the two compatibility equations needed for calculation of stresses and strains in the materials.

Strain compatibility

$\varepsilon_A = \varepsilon_B$

Therefore

$$\frac{\sigma_A}{E_A} = \frac{\sigma_B}{E_B}$$

$$\frac{\sigma_A}{\sigma_B} = \frac{E_A}{E_B} \qquad 1.5$$

Summation of forces

$$F = Force\ in\ MA + Force\ in\ MB = \sigma_A A + \sigma_B B \qquad 1.6$$

Where A = The cross-section area of member MA and B is the cross-section area of member MB

$\sigma_A$ = The stress in member MA and $\sigma_B$ is the stress in member MBA

***Tutorial examples***

1) Determine the increase in length of a steel rod 3metres long and 30mm diameter when subjected to a tensile load of 120kN (E= 210kN/mm$^2$)

*Ans – 2.43mm*

2) A mass concrete pier is subject to an axial load of 2500kN. The pier cross-section is 650mmx800mm and is 2.2metres high. Determine the stress at the base and the amount of shortening that is to occur in the pier. The density of the concrete is estimated as 2,200kg/m$^3$ and the Young's modulus of concrete is estimated as 13 kN/mm$^2$.

3) A short reinforced column is 450mm square and contains four reinforcing steel bars of 25mm diameter. Determine the stresses in the steel and concrete when loaded with 1500kN axial load. $E_s$ = 210 kN/mm2, $E_c$ = 14 kN/mm2.

*Ans – $\sigma_c$ = 6.52 N/mm2 and $\sigma_s$= 97.8 N/mm2*

**Chapter two**

# Structural Analyses

## 2.1 Various aspects of forces

### 2.1.1 Force as vector

#### *a) General*

A scalar quantity is defined by a single item of data. These are quantities which can only be associated with magnitude. Such quantities include time, volume, density, mass etc. Vector quantities on the other hand are specified by magnitude and direction. For force acting at a particular point in a structure, it is necessary to know the direction being applied. Forces which are in he same plane are defined as coplanar and defined by a minimum of two items of data (i.e. one item each of magnitude and direction). Non coplanar forces have to related to three dimensions and require two items of direction in addition to on item of quantity

In a majority of structures it is conventional to simplify the loading into two dimensional problem and hence coplanar. It is usual to relate directional data to specific reference directions such as horizontal or vertical or longitudinal axis of members. The conventional Cartesian axis x and y are employed.

***b) Force components and additions***

In order to analyze forces, more often than not it is convenient to consider the effect of forces in a certain direction. The effects of forces in the Cartesian directions are particularly desirable. In Figure 2.1 below the force P is divided into its components. The single force P is fully specified by its components $p_x$ and $p_y$

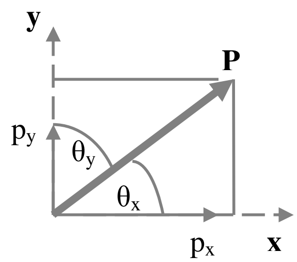



**Figure 2.1 Force components**

$p_x = p\cos\theta_x = p\sin\theta_y$

$p_y = p\cos\theta_y = p\sin\theta_x$

Two coplanar forces may be added by using the parallelogram rule as illustrated in Figure 2.2.

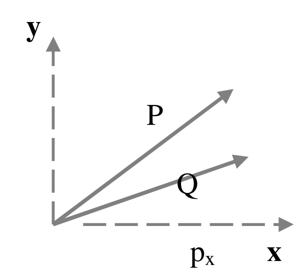
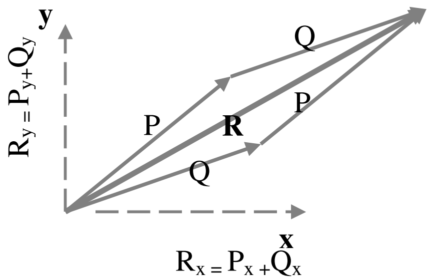


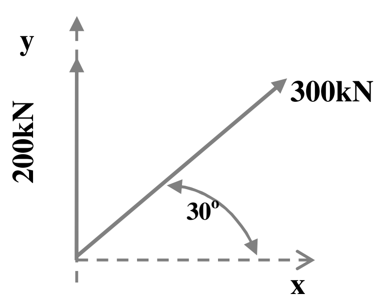

**Figure 2.2 Vector addition**




In the figure the forces P and Q are added to get one force R in magnitude and direction which can be obtained graphically. The vector components of R are the addition of vector components of Q and P. in the use of parallelogram rule the addition of the forces is interchangeable i.e. P+Q = Q+P .

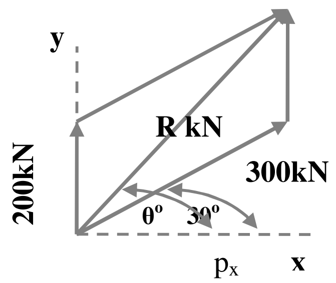
**Example 2. 1**

Add the forces shown on Figure shown below




*Solution -*

Graphically




By calculation

$R_y = 200+300*Cos\ 60^o = 350kN$

$Rx = 200*Cos\ 90^o + 300Cos\ 30^o = 260kN$

$R = \sqrt{}\ (350^2+260^2) = 436kN$

$Tan\ \theta = 350/360 = 1.34$

$\underline{\theta = 53^o}$

### c) *Moments of Forces*

The moment of force is defined as the turning effect of that force. From Figure 2. 3 the force P will cause rotation of the arm AB about A the product of the force and the lever arm l is termed as the moment of the force. The units of moment are force x distance (kilonewtons x metres) kNm.

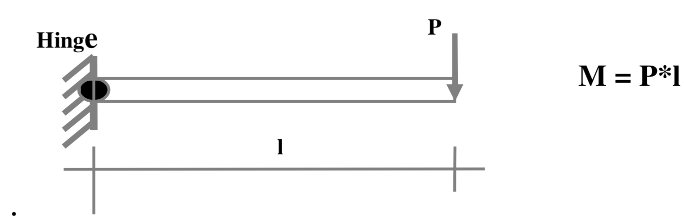



**Figure 2. 3 Moment of a force**

### *d) Reactions, equations of equilibrium, and free body diagrams*

***Reactions***

Structural components are usually held in equilibrium by being secured to fixing points. These fixing points are usually other parts of the structure. The tendency of the forces to cause elements to move is resisted by reactions at the fixing points. The magnitude and direction of the reactions depend on the forces and the types of supports. There three types of supports namely – Pinned, roller pinned and fixed supports (Figure

***Pinned supports*** are those which are held in a particular location. They can not move. Additionally they are only apple to transmit forces in any direction. At a pinned support the moment of the structure forces equals zero. The reaction in a given pinned support is a force in any direction.

***Roller Pinned supports*** are those which are able to roll in given direction only. They can not take load in the direction of motion. Additionally they are only apple to transmit forces a direction perpendicular to the direction of motion. At a pinned roller support the moment of the structure forces equals zero. The reaction at a pinned roller support is a force in a given direction

***Fixed supports*** are those which are held in all directions. They can not move and can not rotate. They are capable of transmitting forces and moments in any direction. The reactions at fixed supports are forces and moments.

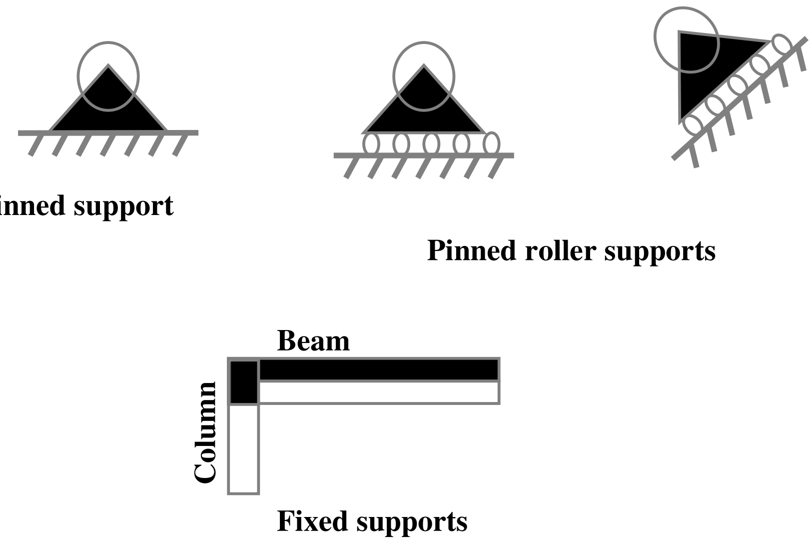



**Figure 2. 4 Different types of supports**

***Equations of equilibrium***

A body is said to be in a state of equilibrium when there is no tendency for it to be disturbed in its present stet of rest or state of uniform motion. A system of forces acting on body which is in equilibrium must have a resultant of all forces equal to zero. For structures the following laws of static equilibrium apply

a) The algebraic sum of vector components of all applied forces referred to any given direction must equal to zero

b) The algebraic sum of the moments of all applied forces about any given point must equal to zero

For a coplanar system of forces these two statements can be summarized as follows

| | |
|---|---|
| *Sum of vertical forces and components = 0* | $\Sigma V = 0$ |
| *Sum of horizontal forces and components = 0* | $\Sigma H = 0$ |
| *Sum of moments about any point = 0* | $\Sigma M = 0$ |

The analysis of all structures is based on the fact that the structure is in equilibrium under the actions of load. The magnitude of reaction area such that the applied loads are exactly counteracted and resisted in accordance with Newton's Law which says actions and reactions are equal and opposite. Furthermore any part of the structure is in a state of equilibrium.

For ease of analysis the structures are divided into elements for analysis purposes. The forces are broken into vertical and horizontal components. The analysis then proceeds by application of the three equations of static equilibrium shown above.

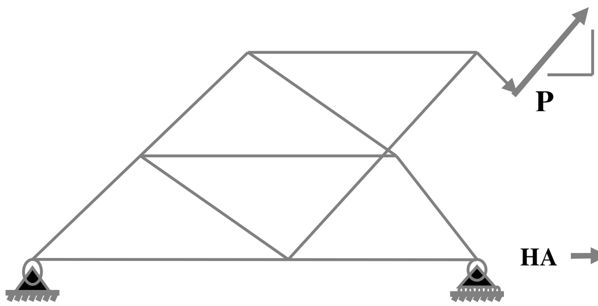
***Free body diagrams***

Free body diagram is a representation of force on a structure. In analyzing a structure a description and estimation of forces that act on a body or part of the body is required. In theory of structures we isolate various components of the structure. Once the component is isolated, all the forces that act on it are drawn. All the forces that act on the part of the structure by external bodies are shown. The isolation of the structural components and representation of the forces in equilibrium is the process of drawing a free body diagram. The free body diagram is the representation of the forces on the structure. The construction of the free body takes the following steps

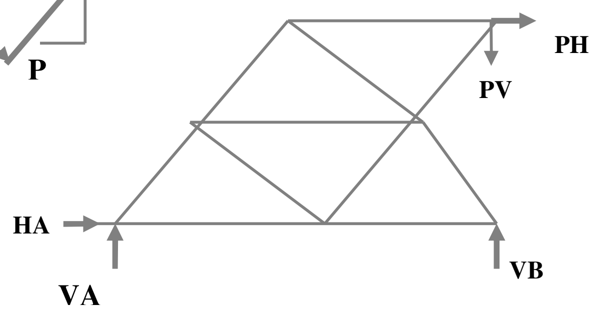
1) A clear understanding of the what the body or combination to be analyzed is required
2) Isolate the body and its external boundary
3) All forces that act on the body are then represented in their proper magnitude and direction. The forces due to self load are also included. Unknown forces should be shown in the assumed direction. After analyses a negative sign shows that the force is in the opposite direction.
4) The choice of coordinates should be indicated in the drawings.

## Example 2.2

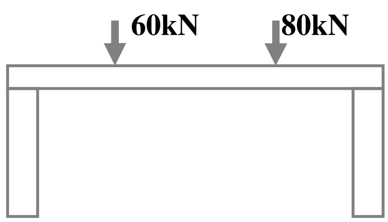
Sketch the free body diagrams for the structures shown below:-



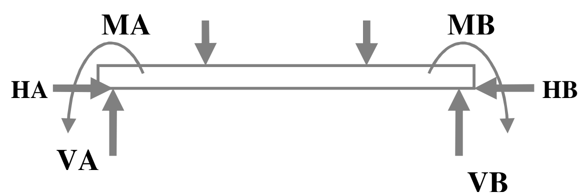

Body ( a pin jointed structure




Free Body




Body ( A beam column system)



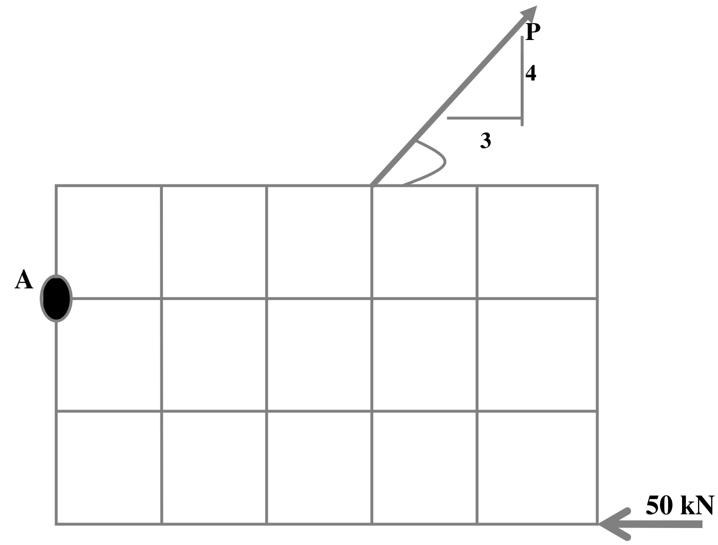

Free Body

**Example 2.3**

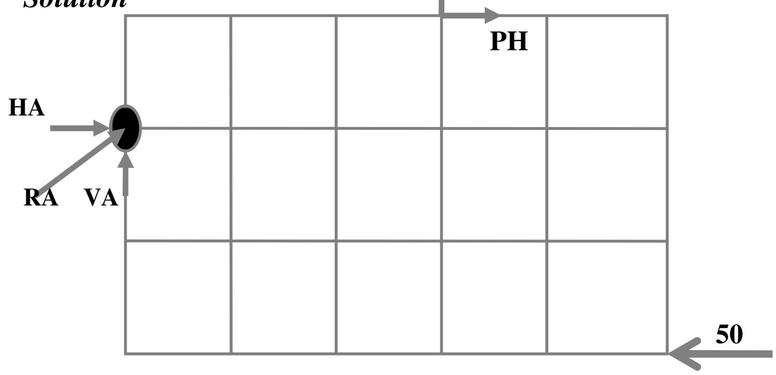
With respect to the sheet of cardboard shown below, calculate

i) The reaction and magnitude of the reaction at A.

ii) The balancing force P




***Solution***




| | | | |
|---|---|---|---|
| (↑) | $\Sigma V = 0$ | $PV + VA = 0$ | ***1*** |
| (→) | $\Sigma H = 0$ | $PH + HA - 50 = 0$ | *2* |
| MA | $\Sigma MA = 0$ | $-PV*3 + PH*1 + 50*2 = 0$ | *3* |
| | | $-P*4/5*3 + P*3/5*1 + 50*2 = 0$ | |
| | | $P = 100*5/9 = 55.56$ kN | |
| | | *PH =P*3/5 =100*5/9*3/5 =33.33* | |
| | | *PV =P*4/5 =100*5/9*4/5 =44.44* | |

1

$P*4/5 + VA = 0$

$VA = -P*5/4 = -100*5/9*4/5$

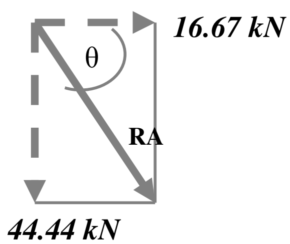
$= -44.44kN$

2

$P*3/5 + HA - 50 = 0$

$HA = 50 - P*3/5 = 50-100*5/9*3/5$

$= 16.67$ kN




$RA = \sqrt{(44.44^2+16.67^2)}$

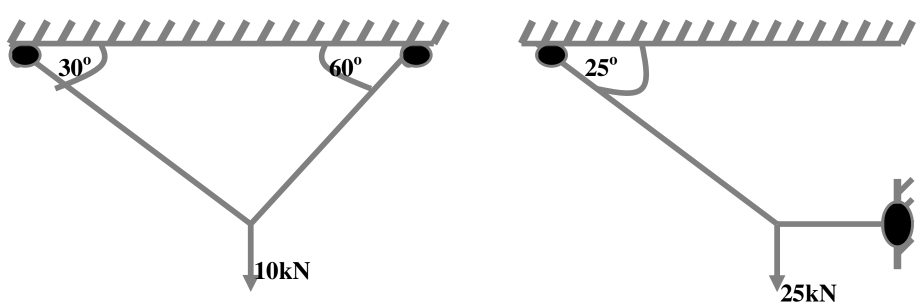
$= 47.46$ kN

$Tan\ \theta = 44.44/16.67 = 2.67$

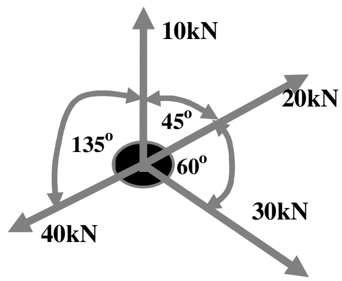
$\underline{\theta = 69.43^o}$

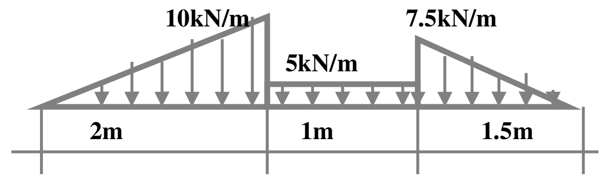
***Tutorial examples***

1) Determine the tension in the wires for the arrangement of loads shown below

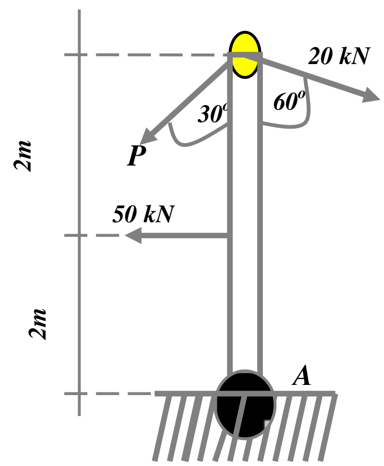


2) Determine the magnitude and direction of the forces shown below




4) Calculate the reaction and its direction at A for a post loaded with point loads at different angles as shown below




## 2.2 Pin jointed structures

### 2.2.1 Introduction

Pin jointed structures range from the very simple coplanar trusses to the exceedingly complex systems consisting of three or more straight or curved members capable of transmitting loads. With increasing accuracy in structural analysis it is now possible to cover large spans over roof tops with three dimensional complexes of pin jointed structures. This has made it possible to cover large sitting areas in stadiums, shopping malls, auditoriums etc.

For the routine designer and for the purposes of analysis some simplification is necessary to avoid lengthy and complicated calculations. This is done by placing the pin jointed structures in two distinct categories as follows:-

i) Statically determinate frameworks in which the magnitude and direction of forces acting on the members may be determined by direct application of the laws of static equilibrium

ii) Statically indeterminate frameworks in which the number of unknown quantities exceeds the number quantities determinable by application of the laws of static equilibrium. The consideration of these latter frameworks which include portal frames, rigid jointed frameworks with members in bending and or in torsion is beyond the scope of this course.

The conditions needed for the statically determinate pin jointed structural analyses covered in this course are as follows:-

i) The members are straight, inextensible and weightless

ii) The members are connected at their extremities to other joints by means of pin joints which function as perfect hinges (i.e.) they are free to rotate and can transmit no moment.

iii) All members, applied forces, and reactions lie in the same plane

iii) The framework is capable of resisting geometrical distortion under any system of forces applied at the nodes.

The above conditions can not be met by practical frameworks. Most frameworks analyzed using the above conditions are usually gusseted, nailed without the prescribed hinges. However the assumptions enable analysis and design of safe pin jointed frameworks

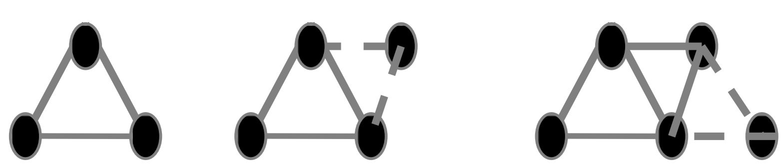
### 2.2.2 Analysis of statically determinate frameworks

A framework which is statically determinate contains the correct number of members required to keep it stable. This frame is also termed as a perfect frame. Frameworks having fewer members than required number are unstable and referred to as sub static. Frameworks with more members than a sufficient number are hyper static and contain redundant members which can be removed without producing instability. Figure 2.5 shows the simplest possible framework that would satisfy the required conditions of analysis. The relationship between the member of members and the number of joints is given by Equation 2.1.

$$m = 2j - 3 \qquad \textbf{2. 1}$$

Where m = number of members

J = number of joints



| | | | |
|---|---|---|---|
| Members | 3 | 5 | 7 |
| Joints | 3 | 4 | 5 |

**Figure 2. 5** Statically determinate pin jointed frameworks

If the more than one joint is pinned to a rigid support, the number of members required to make a perfect frame is given by Equation 2.2. Figure 2.6 demonstrates the use of Equations 2.1 and 2.2. The figure shows examples of perfect, unstable and hyper static frameworks

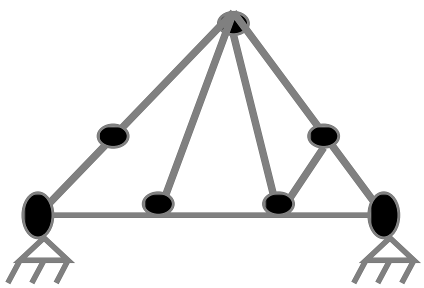
$$m = 2(j - s) \qquad \textbf{2. 2}$$

Where m = number of members

j = number of joints

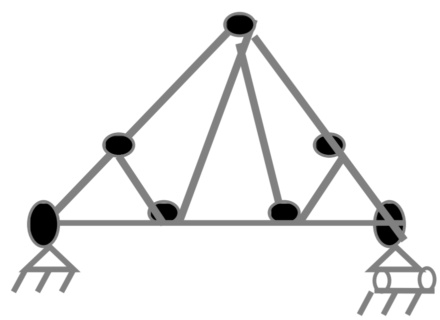
s = number of joints pinned to a rigid external support

**_m= 2(j-s)_**



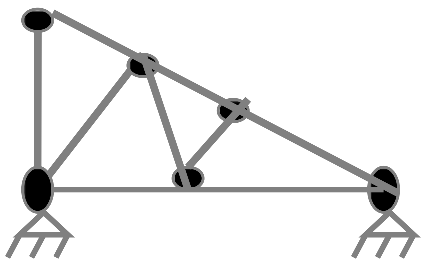
m=11
2(j-s)=2(7-2) =10
m>2(j-s)
Therefore hyper static frame

**_m= 2j-3_**

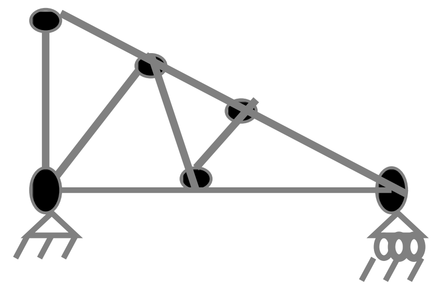


Roller allows movement therefore this support is not rigid
m=11
2j-3=2*7-3 = 11
m=2j-3
Therefore perfect frame

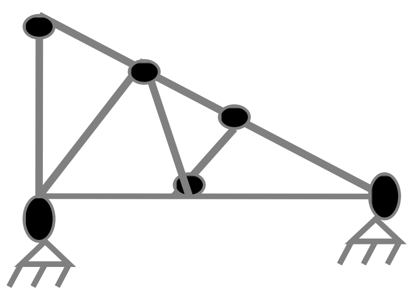
***m= 2(j-s)*** ***m= 2j-3***



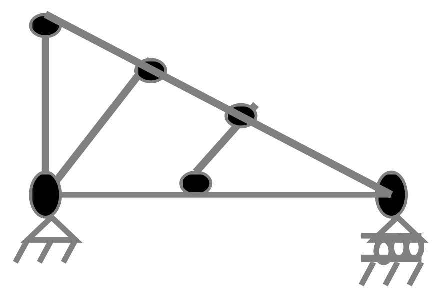
m=9
2(j-s)=2(6-2) =8
m>2(j-s)
Therefore frame is hyper static



m=8
2j-3=2*6-3 = 9
m<2j-3
Therefore unstable frame



m=8
2(j-s)=2(6-2) =8
m=2(j-s)
Therefore perfect frame



m=9
2j-3=2*6-3 = 9
M<2j-3
Therefore a perfect frame

0

**Figure 2. 6 Perfect, unstable and hyper static frameworks**

### 2.2.3 Methods of analysis

The solution of the forces acting in the members in the frames is usually done by the application of the three equations of static equilibrium. Two such methods are considered below

i) Method of resolution

- Obtain the reactions for the entire structure

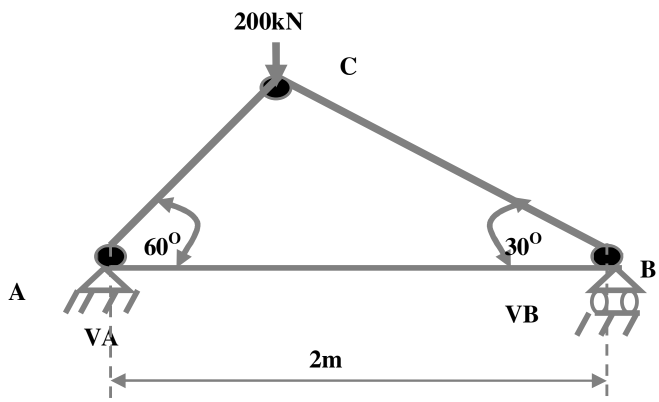
- Consider the forces acting on individual joints starting from not more than two forces are unknown. Application of the equations of static equilibrium for each joint yields the unknown forces.
- Proceed from the known forces in members to the joints with unknown forces. In general proceed to joints with one or two unknown forces.
- In the analysis assume that the forces in the members are in tension. In the event the analysis yielded a negative value the force is compression

ii) Method of sections

- Obtain the reactions for the entire structure
- Make a theoretical section cutting the entire structure. The members whose forces are needed should also be cut. A maximum of three unknowns should be cut.
- Each of the sections of the structures is analyzed using the three equations of static equilibrium to give the value of the unknown forces
- In the analysis assume that the forces in the members are in tension. In the event the analysis yielded a negative value the force is compression

**Example 2. 4**

Determine the reactions and the forces in all the members of the pin jointed structures shown in figure




i) Method of resolution

***Entire structure***

↑ VA + VB – 200 = 0     1)

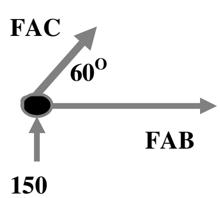
A VB*2 + 200*1cos 60=0     2)

VB = 100/2 = 50kN

B VA*2 -200*1.5=0     3)

VA = 200/1.5 = 150kN

***Joint A***



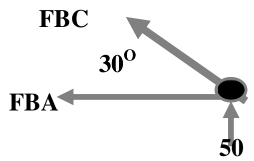

↑ FAC*sin 60 + 150 = 0     1)

***FAC=-150/sin60*** **=*-173 kN* (Compression)**

→ FAC*cos60 +FAB = 0     2)

FAB**=*-FAC*cos60*** **=*--173*cos60 kN =86.6kN* (Tension)**

***Joint B***




FBC*sin 30 + 50 = 0 1)

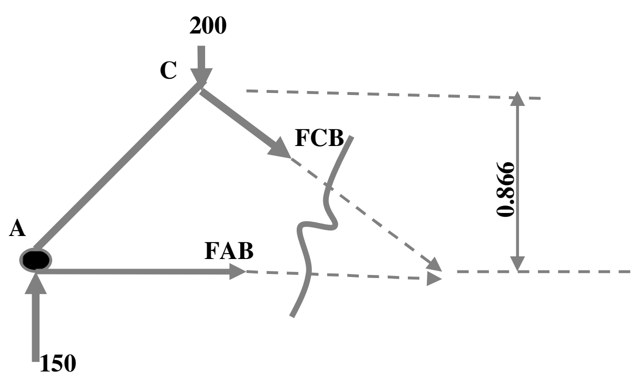
***FBC=-50/sin30 =-100 kN (*Compression)**

-FBC*cos30 -FBA = 0 2)

***FBA=-FBC*cos30 =--100*cos30 kN =86.6kN (Tension)***

Therefore checks

## *ii) Method of sections*




**C** 150*0.5 –FAB*0.866 =0 1)

FAB =150*0.5/0.866

***= 86.6 kN (Tension)***

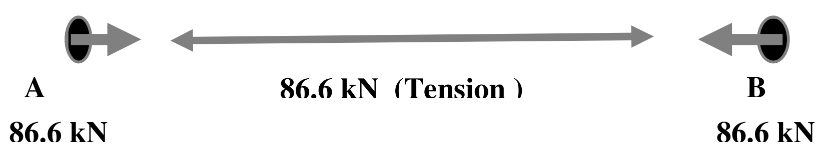
**A** (200+FCB*cos60)*.5+FCBcos30*.866 =0 2)

100+.25FCB+.75FCB =0

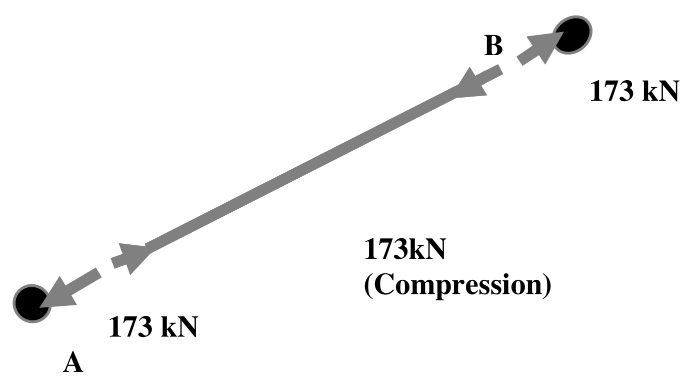
***FCB=100/1 =-100 (Compression)***

### *Type of forces analyzed*

The forces analyzed above are acting on the joints. The forces in the members are equal and opposite to those acting on the joints. Examine the forces in joints A and B the figure below. The member is acted upon by tensile force of equal magnitude by application of the law that action and reaction are equal and opposite. By similar examination of force on joints A and C the force in Member AC is seen to be in compression.




**Force in Member AB**

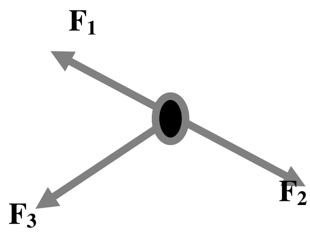



**Force in Member AC**

### *Two simple rules in the analysis of pin jointed structures*

a) When a joint has 3 members and no external force and two members are in straight line as shown on Figure

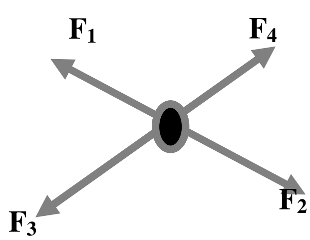
- The forces in the straight members are equal
- The force in the third member is equal to zero




$$F_1 = F_2$$
$$F_3 = 0$$

b) When a joint has four members and no external force and two opposite members are in straight line as shown on Figure

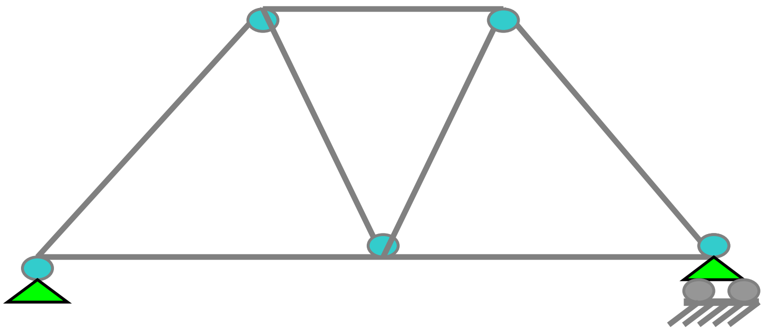
- The forces in the straight members are equal

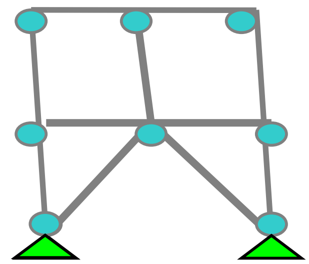


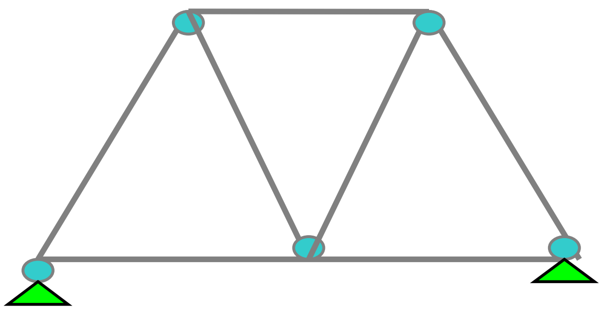

$$F_1 = F_2$$
$$F_3 = F_4$$

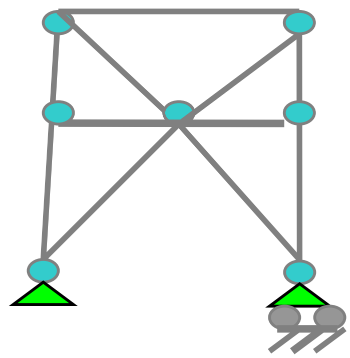
***Tutorial examples***

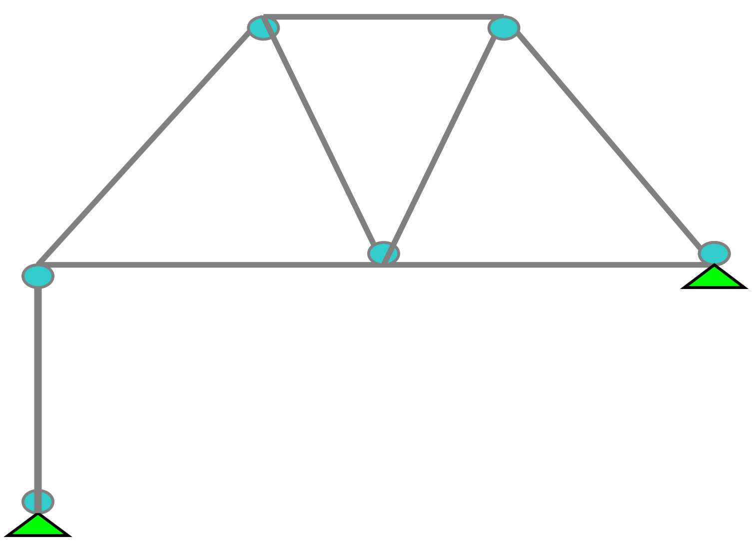
1) For the following pin-jointed structures determine whether the structures are unstable, perfect or redundant







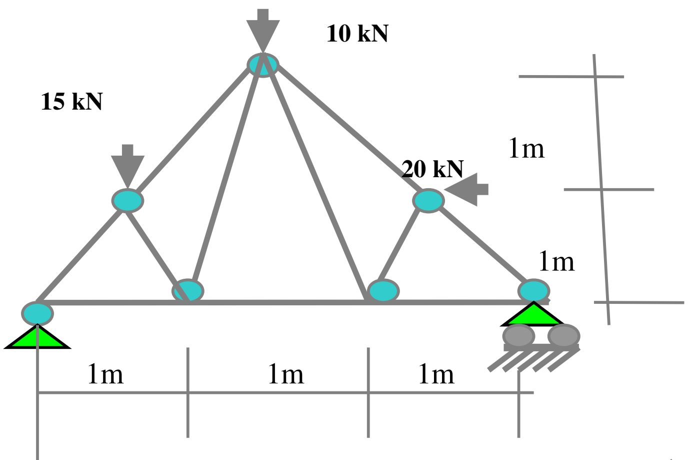


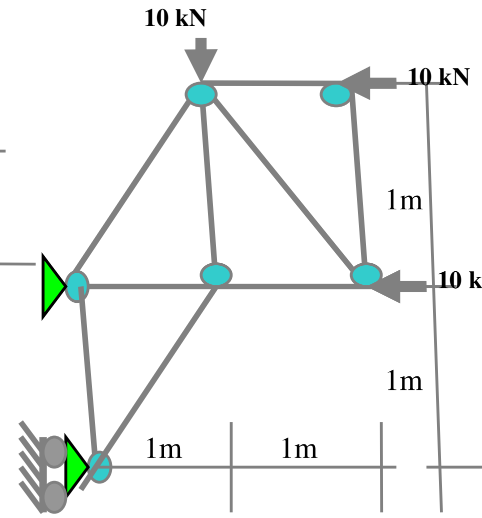


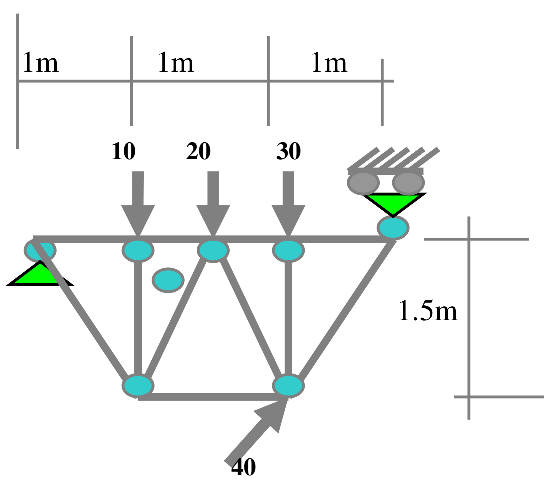
2) For the following pin-jointed structures determine

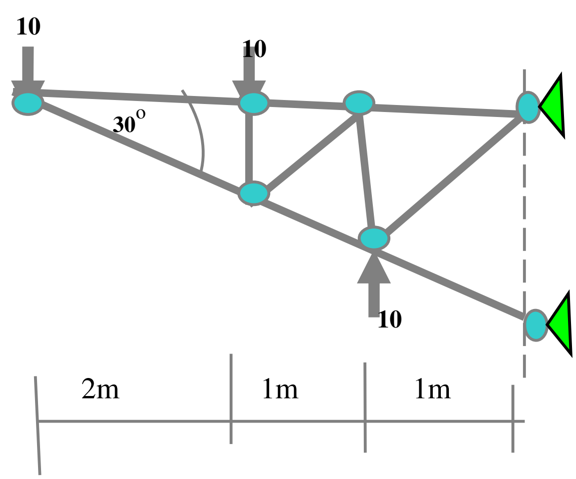
i) The reactions in magnitude and direction

ii) The forces in all the members



## 2.4 Analysis of parallel forces and simply supported beams

### 2.4.1 Equivalent force of parallel forces

The addition of parallel forces can be demonstrated with the forces shown on Figure 2.

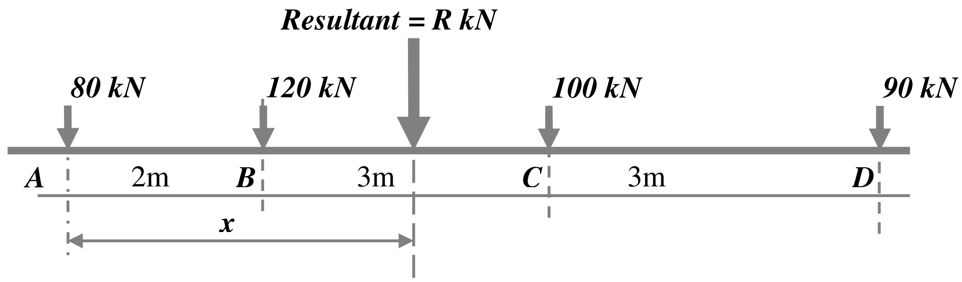



**Figure 2. 7 Addition of parallel forces**

The equivalent force R has the same effect as the individual forces, 80, 120, 100 and 90 kN. This can be summed up as follows.

- The sum of vertical components of the forces must equal to the vertical component of the resultant
- The sum of horizontal components of the forces must equal to the horizontal component of the resultant
- The sum of moments of the forces about any must equal to the horizontal component of the resultant

Since all the forces applied are vertical

R = ΣV

R = 80+120+100+90 =390kN

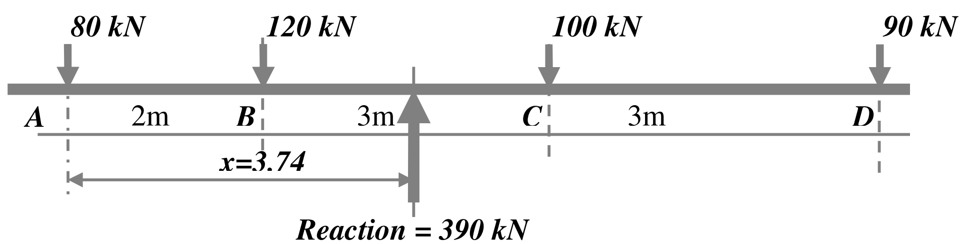
A :R*$x$= 120*2+100*5+90*8

$x$ =1460/390 = 3.74m

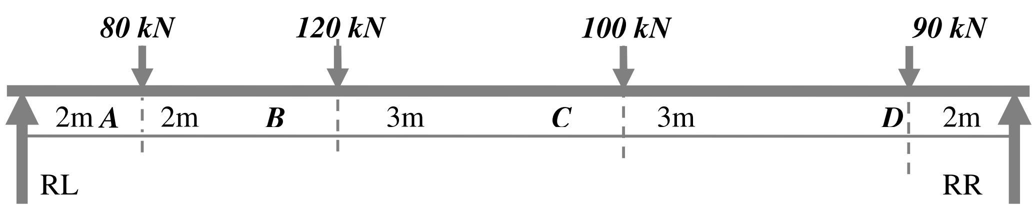
### 2.4.2 Beam reactions

If the forces shown in the previous example were to be applied to a horizontal beam then the beam could be held in equilibrium by a single force (Reaction) which is equal and opposite to the resultant R Figure 2. 8a. However such a system would be unstable because at the slightest imbalance the beam would tilt. A better arrangement is to have at least two reactions as shown in Figure 2.8b.

This is now a structural member and the reactions on the left (RL) and that on the right (RR) must balance the forces applied on the beam and the three equations of static equilibrium must be applicable.




*a) Single reaction*




*b) Two reactions*

**Figure 2. 8 Beam reaction to parallel forces**

The solution to the beam supported on two supports would be as follows

$$\Sigma V = 0: \quad RL + RR - 390 = 0$$

$$\boldsymbol{RL + RR = 390}$$

**RL =0** -RR*12+80*2+120*4+100*7+90*10=0

***RR = 2240/12 =186.7kN***

**RR=0** RL*12-80*10-120*8-100*5-90*2=0

***RL = 2440/12 =203.3kN***

***Checking***

Vertical forces equal to zero

***RL +RR=186.7+203.3 = 390***

The moment for the entire structural system is zero about any point

**A=0** 203.3*2+120*2+100*5+90*8-186.7*10=0

### 2.4.3 Shearing forces and bending moments

Previously we have shown that the loading of a beam results in deformation in the form of bending. The primary load effects being experienced by the beam are shearing force and bending moment. These primary effects are now considered in detail.

When a beam is loaded laterally, it must transmit to its supports some proportional of the applied load. This is done by development of the shearing force in the beam fabric. The beam also develops resistance to bending for it to remain straight. These two load effects can be illustrated by a beam built into a support at a point A also known as a cantilever beam (**Figure 2.9**). The cantilever beam transmits to the support a reaction R. At any given section before the support is reached the beam must transmit the load W. This is done by the beam fabric and development of shearing force $Q_x$. failure of the beam to develop sufficient shearing resistance will cause the beam to split as shown in **Figure 2.9**. In addition to the split effect the beam will bend. The resistance to bending is resisted by bending moment developed in the beam fabric.

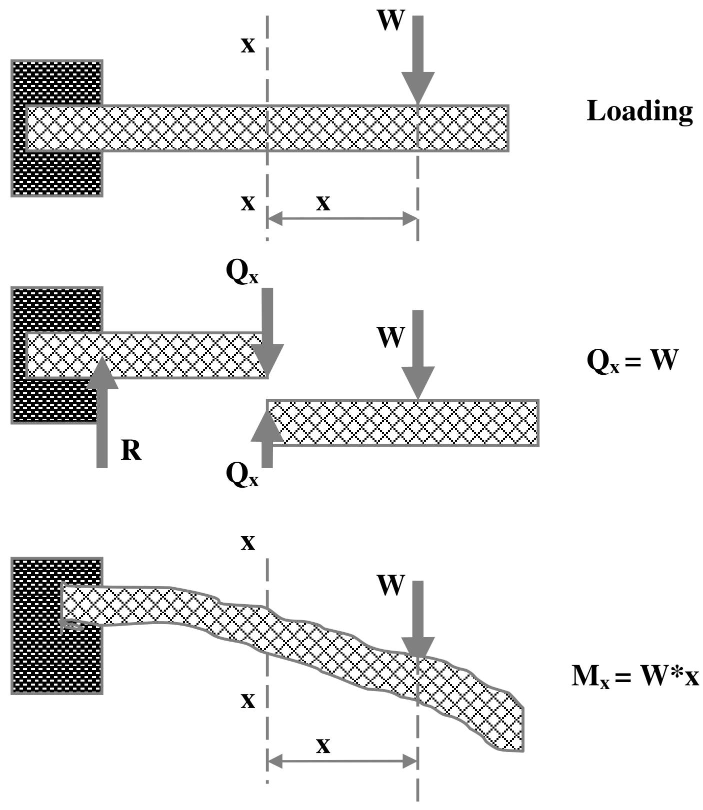



**Figure 2.9** Development of shearing force and bending moment in a cantilever beam

The shearing force and the bending moment transmitted across the section x-x may be considered as the force and moment respectively that are necessary to maintain the beam in a serviceable condition at x-x. The shearing force at x-x is W while the bending moment at x is W*x. In a nutshell the definition of these two fundamental load effects can be defined as follows:

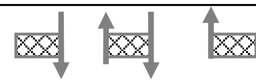
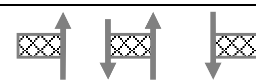
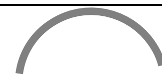
*Shearing force* is the algebraic sum of all the forces acting **on one side of the section.**

*Bending moment* is the algebraic sum of all the moments at the section of all the forces on **one side of the section**.

***Sign convention***

The sign convention of the shearing forces and bending moments is shown on Table 2. 1

**Table 2. 1 shearing and bending moment conventions**

| **Load effect** | **Symbol** | **Sign convention** | | **Units** |
|---|---|---|---|---|
| | | **Positive** | **negative** | |
| Shearing force | Q or S |  **down on left** |  **up on left** | kN |
| Bending moment | M |  **Sagging – tension at the** |  **Hogging – tension on top** | kNm |

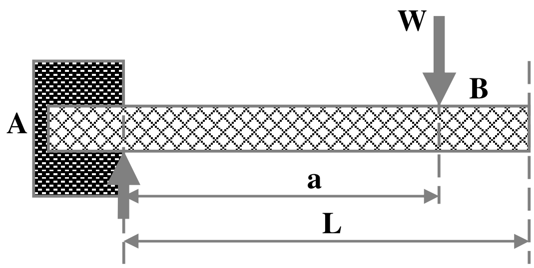
Diagrams showing the variation of shearing force and bending moment are usually drawn in the design process. This enables design of the various sections of the beam. The diagrams are usually drawn as follows:

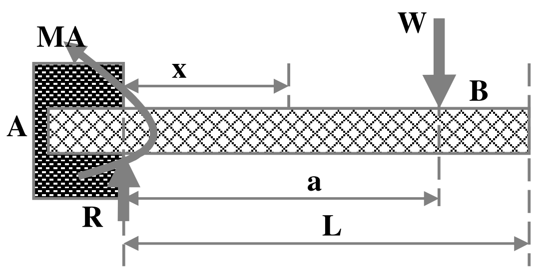
- The beam is considered as the base of the effects
- The ordinates are the effects
- Draw the shearing force diagram (SFD) directly below the loading diagram and the bending moment diagram (BMD) directly below the SFD.
- Use the sign rule for the SFD. This is easiest done by starting the drawing from the left hand side and plotting in the direction of the force
- The bending moment is best plotted with positive values on the tension side of the beam. The bending moment takes the shape of the deflected shape of the beam.

**Example 2. 5**

Draw the shearing force and the bending moment of the following beams

a) A cantilever beam with a point load.




***Free body diagram***

***Reactions***

$\Sigma V = 0$: R-W =0

***R =W***

A -MA+W*a =0

**MA = Wa**

***Variation of shear and moment***

At x<a Qx = W

Mx = -w*(a-x)

At x=0 Qx = W

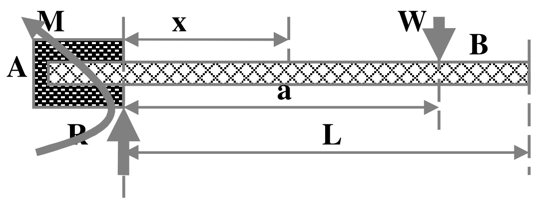
Mx = -w*a

At x>a Qx = 0

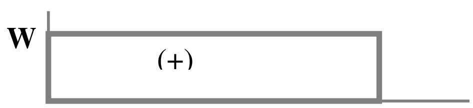
Mx = 0

| **section** | **Qx** | **Mx** |
|---|---|---|
| x<a | W | -w*(a-x) |
| x>a | 0 | 0 |
| X=0a | W | -w*(a) |

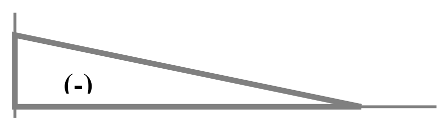
*Drawing of loading, shear and bending moment diagrams*




*Load diagram*

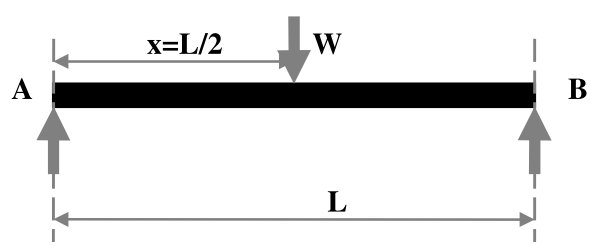


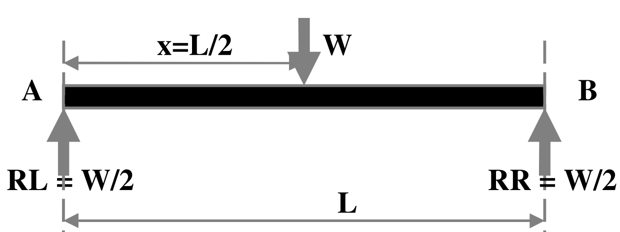

*Shearing force diagram*




*Bending moment diagram*

b) Beam with central load at the middle



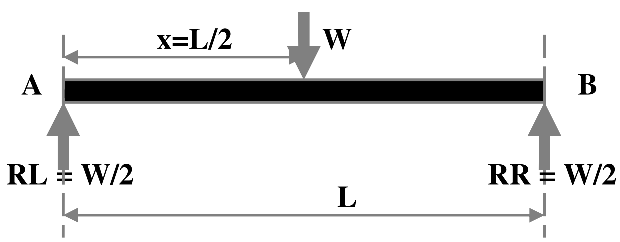

*Free body diagram*

*Reactions*

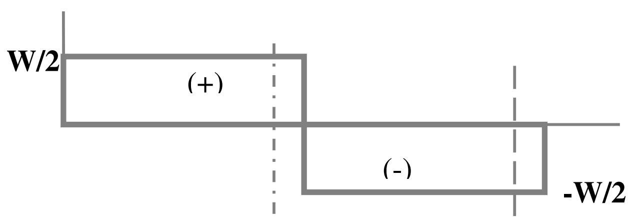
By inspection RL = W/2 and RR = W/2

*Variation of shear and moment*

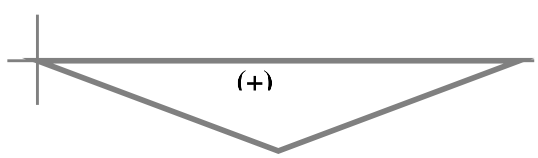
| Section | Qx | Mx |
|---|---|---|
| x<W/2 | W/2 | -W/2*x |
| X=L/2 | 0 | WL/4 |
| Maximum moment = WL/4 | | |




*Load diagram*

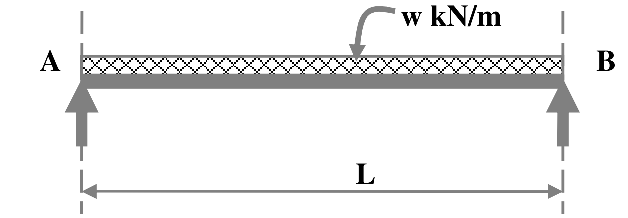



*Shearing force diagram*

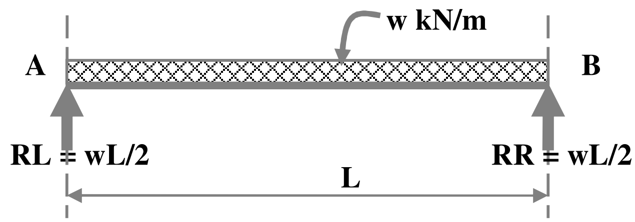



*Bending moment diagram*

## C) Beam with uniformly distributed load




*Load diagram*



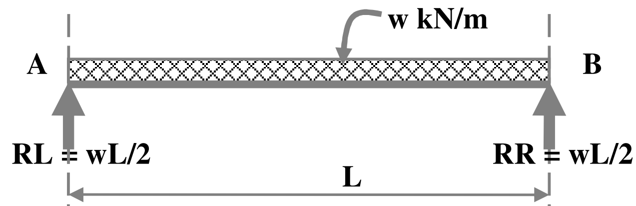

*Free body diagram*

*Reactions*

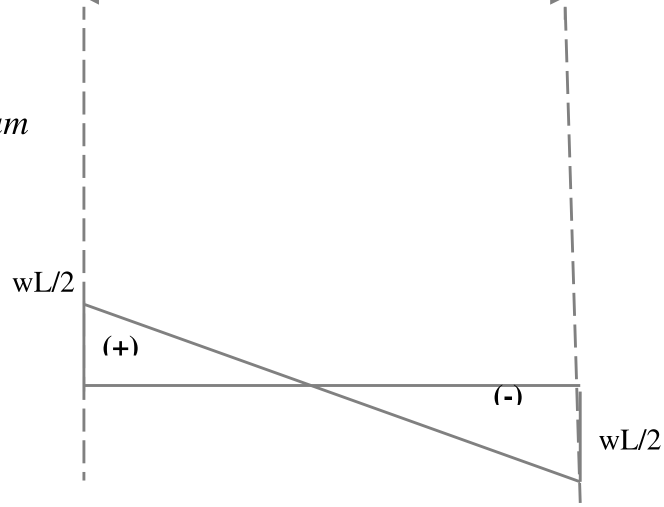
By inspection RL = wL/2 and RR = wL/2

***Variation of shear and moment***

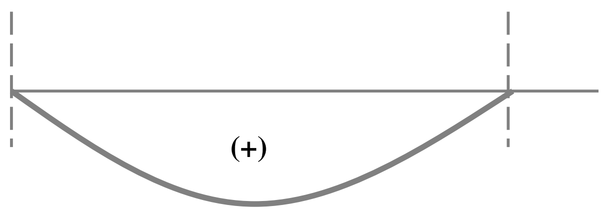
| **Section** | **Qx** | **Mx** |
|---|---|---|
| x | RL-wx | RL*x-w*$L^2$/2 |
| x=0 | RL = wL/2 | 0 |
| Maximum moment = w$L^2$/8 | | |




*Load diagram*



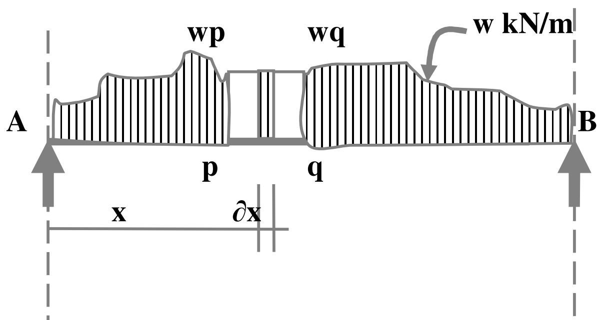

*Shearing force diagram*



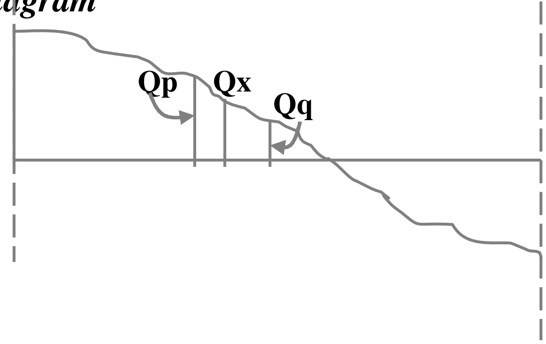

*Bending moment diagram*

## 2.4.5 Relationship of loading, shearing force and bending moment diagrams

Consider an elemental distance $\partial x$ on the beam (Figure 2. 10). The load on the element is $w\partial x$. the element is bound by sections at p and q.




***Load diagram***




***Shearing force diagram***


***Bending moment diagram***

**Figure 2. 10 variable load, shearing force and bending moment for a beam**

For the element (Figure 2.11) within the beam, the sum of the vertical internal forces generated within the beam fabric must equal to zero for internal stability (Equation 2.3). In addition the sum of the moments about any point must equal to zero (Equation 2.4). The application of these two equations to enable static equilibrium of the element leads into the relationship between the loading, the shearing force and bending moment diagrams.

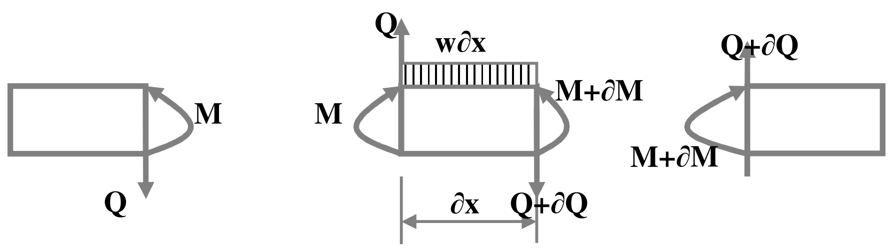



**Figure 2. 11 Element within a beam**

$$\Sigma V = 0 : Q - w\partial x - (Q + \partial Q) = 0 \qquad 2.3$$

$$\partial Q = -w\partial x$$

$$therefore\ \frac{\partial Q}{\partial x} = -w \quad and \quad \int dQ = -\int wdx$$

Integrating between the section of the beam bound by p and q gives the change in the shearing force between the two points. The result of the above integration can be summarized as follows:-

***The change in shearing force between two points on a beam is equal to the area under the loading diagram between these two points***

$$\int_p^q dQ = Qq - Qp = -\int_p^q wdx \ and \quad therefore \quad Qp - Qq = \int_p^q wdx$$

Taking moments about the end of the element.

$$\Sigma M = 0 : Q\partial x + M - (M + \partial M) - w\partial x * \partial x/2 = 0 \qquad 2.4$$

Now $w\partial x*\partial x/2$ is small and can be neglected. The relationship can be simplified as follows

$$\partial M = Q\partial x$$

$$therefore\ \frac{\partial M}{\partial x} = Q \quad and \quad \int dM = Mp - Mq = \int_p^q Qdx$$

As in the above derived relationship of change in shearing force and the loading, integrating between the section of the beam bound by p and q gives the change in the bending moment between the two points. The result of the above integration can be summarized as follows:-

***The change in bending moment between two points on a beam is equal to the area under the shearing force diagram between these two points***

A further relationship of moment and shearing force can be obtained by further analysis of the differential equations as follows

$$Q = \frac{\partial M}{\partial x} \quad and \quad therefore \quad M = \int Qdx = -\int\int wdx^2$$

M=f(x)

The maximum value of M occurs when dM/dx = 0. This occurs when the shearing force is equal to zero. The result can be summarized as follows

***The maximum value of the bending moment occurs on the beam at the point where the shearing force equals zero***

## 2.4 Analysis of simple arches and compound beams

So far we have analyzed pin jointed structures and simply supported beams. The introduction of pins has enables analysis of the pin jointed structures on the basis that they do not transmit moment. The introduction of pins in arches and compound beams also enables the analysis of arches and compound beams as shown in Example 2. 6 and Example 2. 7.

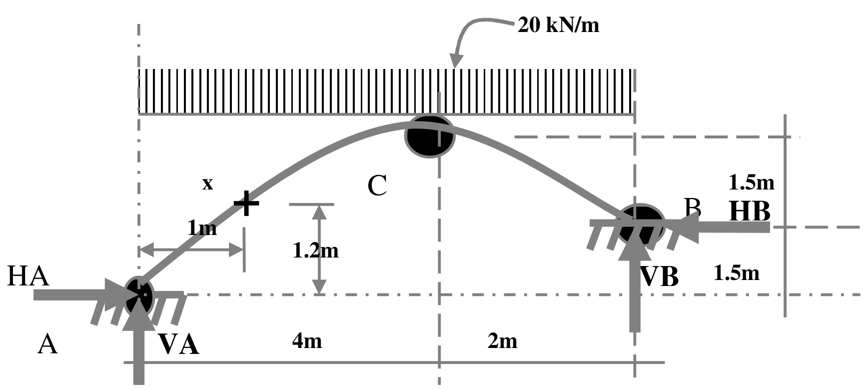
**Example 2. 6 Arch with a pin**

For the arch shown below

i) Calculate the reactions in magnitude and direction

ii) The bending moment at x

iii) The shearing force at x assuming that the slope of the arch at x is $30^o$ to the horizontal



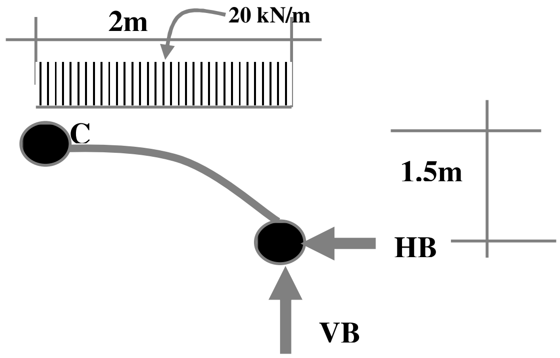

For the entire structure

$$\uparrow \sum V = 0: \quad V_A + V_B - 20 * 6 = 0 \qquad 1$$

$$\rightarrow \sum H = 0: \quad H_A - H_B = 0 \qquad 2$$

$$\curvearrowright \sum A = 0: \quad V_B * 6 - H_B * 1.5 + 20 * 6 * 3 = 0 \qquad 3$$

There are three equations and four unknowns. The reactions can not be determined. The introduction of a pin at C allows separating the structure at C and analyzing the separate parts as follows:-



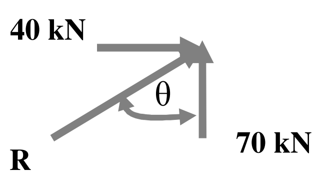

C:- HB x1.5 – VBx2+20x2x2/2 =0 ***4***

From equation 3 and 4:- 8VB+400 =0 VB = 50kN

HB=60/1.5 =40kN

From equation 1:- VA + 50= 120:- VA =70kN

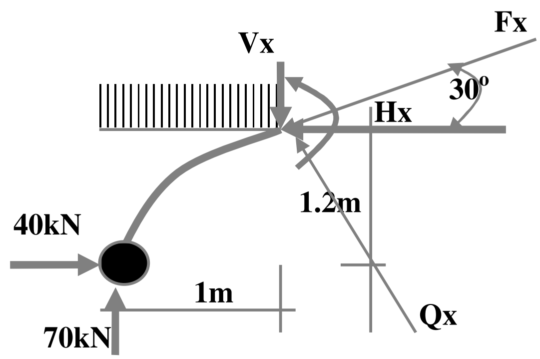
From equation 2 HA = HB =40kN




Reaction at A: - $RA = \sqrt{(40^2+70^2)} = 80.62$

$\text{Tan}\ \theta = 40/70$ therefore $\theta = 29.7^o$

***Moment, axial and shearing force at x***




***Moment at x***

Mx $=70x1.0-40x1.2-20*1^2/2=12$ kNm

***Axial and shearing force at x***

Vx =70-20=50 kN

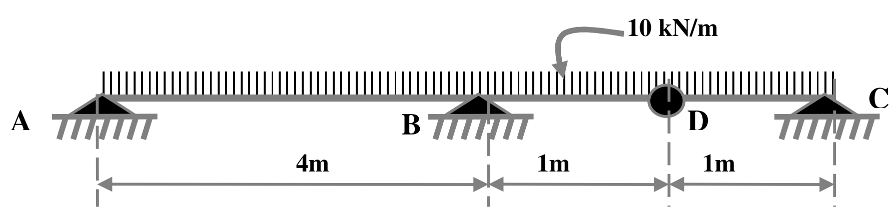
Hx = 40 kN

Qx = 50cos30o-40xcos60o =23.3 kN

Fx =50 cos60+40cos30o =59.64 kN

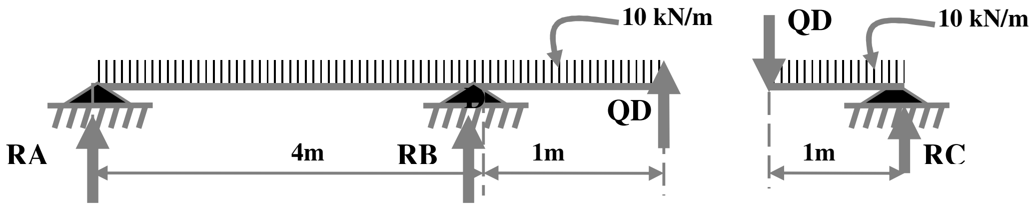
**Example 2. 7 Compound beam with a pin**

Calculate the reactions at A, B and C for the compound beam shown below. The introduction of a pin at D allows to have a point in the beam whose moment is zero. This point can only transmit shearing force. This scenario allows us to analyze the compound beam by separating the beam at the pin and considering the two ends as statically in equilibrium.



***Solution***

Separate the beam at the pin




***Consider right side of the pin***

$\Sigma V = 0$ :- -QD-10*1+RC = 0

D=0 :- -RC*1+10*1*1/2 = 0 RC = 10/2 = 5kN

QD = RC -10 QD = 5-10 = -5kN

***Consider left side of the pin***

B:- RA*4-5*10*1.5+5*1=0 RA = 70/4 =17.5

OR B:- RA*4-10*4*4/2+10*1*1/2+5*1 = 0 RA =70/4 =17.5

A :- -RB*4+10*5*5/2+5*5=0 RB = 150/4 RB =37.5

RA + RB +RC = 5+17.5+37.5 = 60 okay

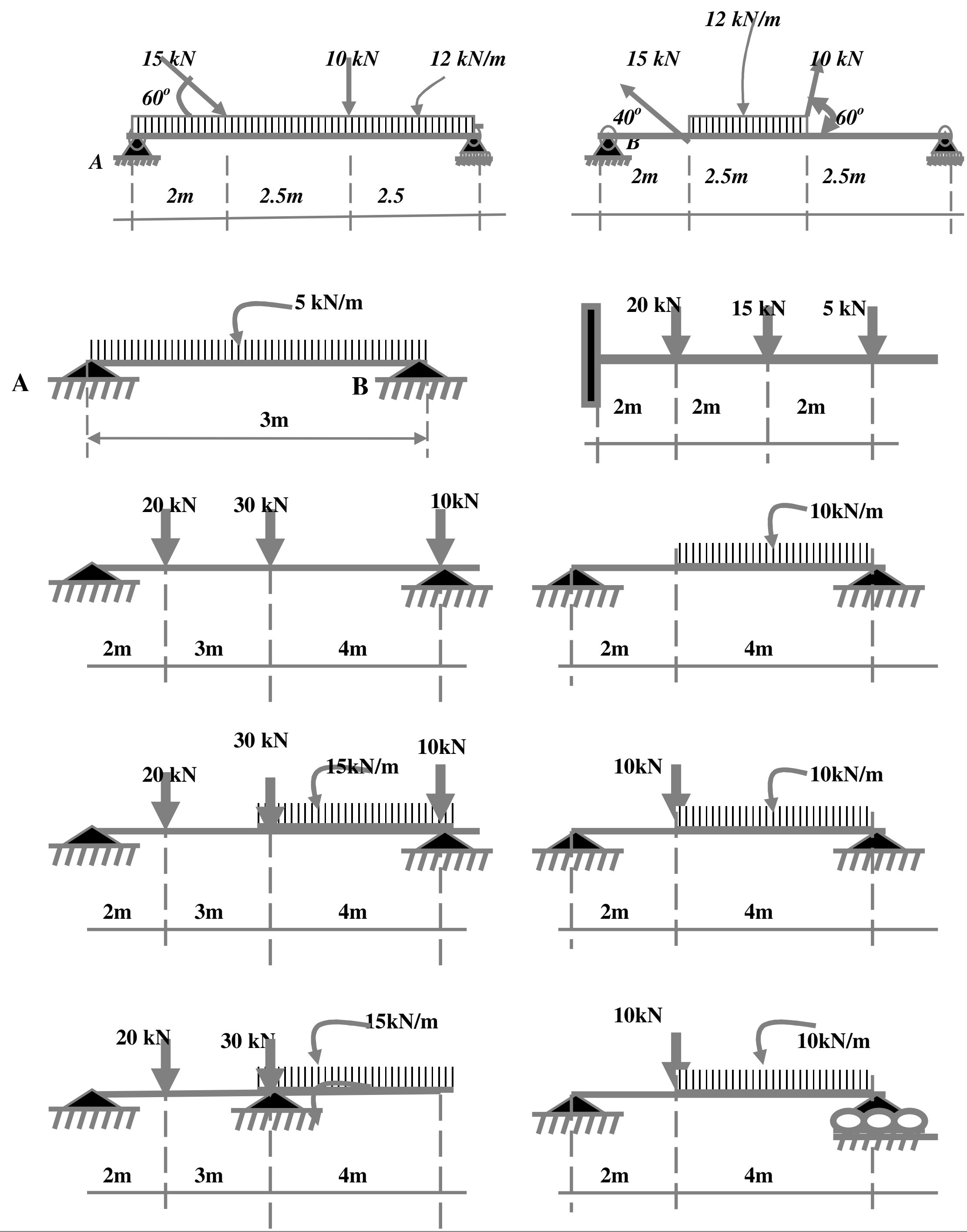
## Tutorial examples

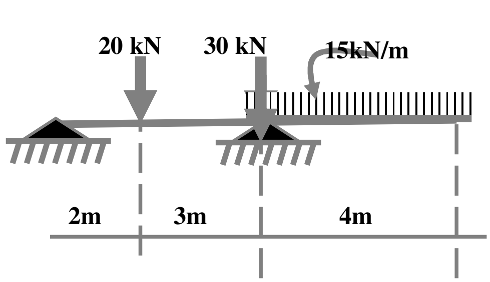
1) For the each of the beams shown below:-

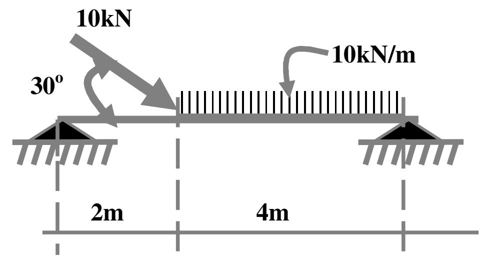
   i) Calculate the reactions in magnitude and direction

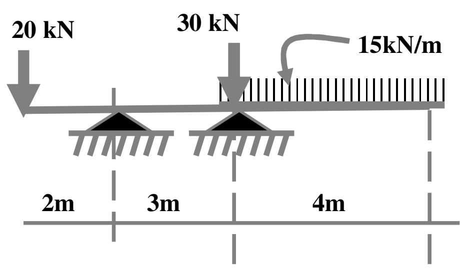
   ii) Draw the shearing and the bending moment diagrams

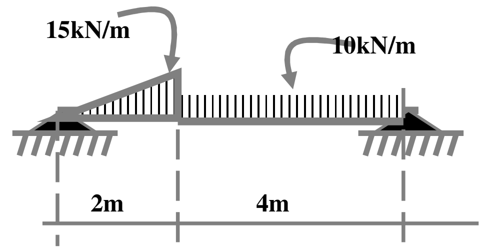
   iii) Determine the magnitude and the position of the maximum bending moments



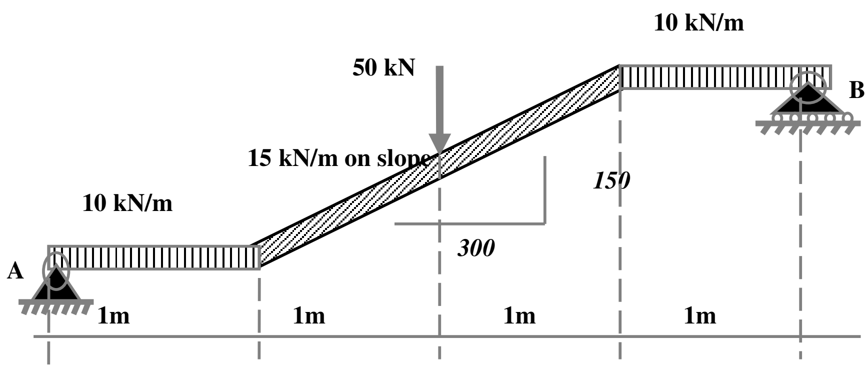


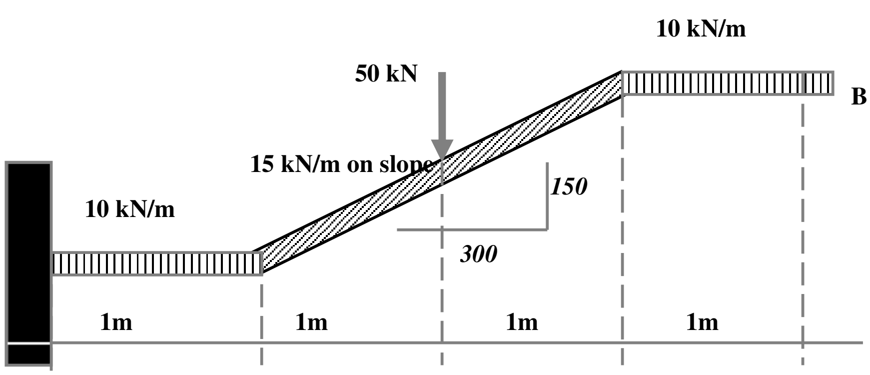




5) Calculate the reaction and its direction for the two staircase sections shown below




i) Normal staircase



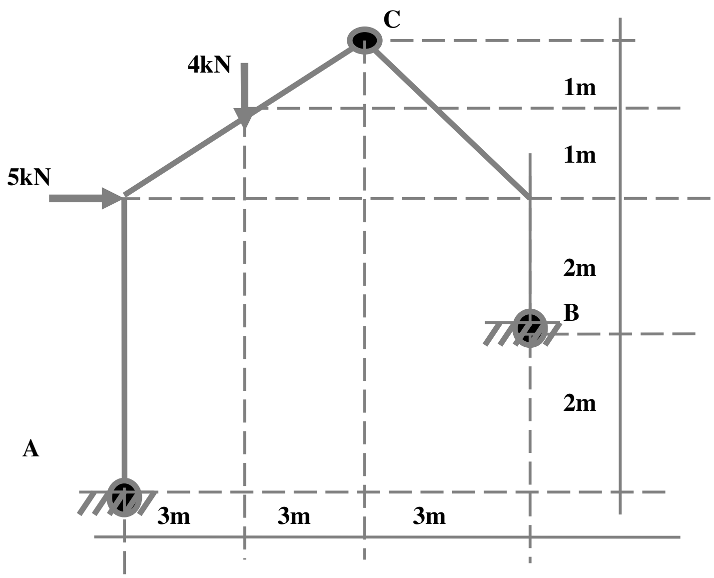

ii) Fixed on one side staircase (cantilevering)

6) Calculate

i) The reactions at A and B for the three pinned arch shown below. Obtain the magnitude and direction.

ii) The bending moment at x

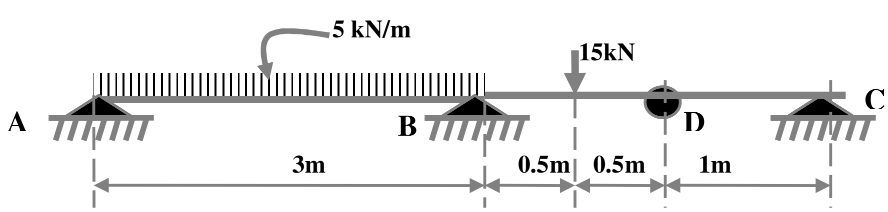
The shear force and the axial force at x




7) For the beam shown below draw

   i) The shearing force diagram

   ii) And the bending moment diagram




***Good luck SK Mwea***